1.2 — Essential Micro Concepts

ECON 316 • Game Theory • Fall 2021

Ryan Safner

Assistant Professor of Economics

safner@hood.edu

ryansafner/gameF21

gameF21.classes.ryansafner.com

Outline

Game Theory vs. Decision Theory

Optimization & Preferences

Solution Concepts: Nash Equilibrium

Game Theory vs. Decision Theory

The Two Major Models of Economics as a "Science"

Optimization

- Agents have **objectives** they value
- Agents face **constraints**
- Make **tradeoffs** to maximize objectives within constraints

Equilibrium

- Agents **compete** with others over **scarce** resources
- Agents **adjust** behaviors based on prices
- **Stable outcomes** when adjustments stop

Game Theory vs. Decision Theory Models I

Game Theory vs. Decision Theory Models I

- Traditional economic models are often called **"Decision theory"**:
- **Equilibrium models** assume that there are **so many agents** that **no agent's decision can affect the outcome**
  - Firms are price-takers or the *only* buyer or seller
  - **Ignores all other agents' decisions**!
- **Outcome**: **equilibrium**: where *nobody* has any better alternative

Game Theory vs. Decision Theory Models III

- Game theory models directly confront strategic interactions between players
  - How each player would optimally respond to a strategy chosen by other player(s)
  - Lead to a stable outcome where everyone has considered and chosen mutual best responses
- **Outcome**: Nash equilibrium: where *nobody* has a better strategy **given the strategies everyone else is playing**

Equilibrium in Games

- **Nash Equilibrium**:
  - no player wants to change their strategy **given all other players' strategies**
  - each player is playing a **best response** against other players' strategies

Optimization & Preferences

Individual Objectives and Preferences

- What is a player's **objective** in a game?
  - "To win"?
  - Few games are purely zero-sum
- "De gustibus non est disputandum"
- We need to know a player's **preferences** over game outcomes

Modeling Individual Choice

- The **consumer's utility maximization problem**:

1. **Choose:** < a consumption bundle >
2. **In order to maximize:** < utility >
3. **Subject to:** < income and market prices >

Modeling Firm's Choice

- 1st Stage: firm's profit maximization problem:

1. **Choose:** < output >
2. **In order to maximize:** < profits >

- 2nd Stage: firm's cost minimization problem:

1. **Choose:** < inputs >
2. **In order to *minimize*:** < cost >
3. **Subject to:** < producing the optimal output >

Preferences I

- Which game outcomes are **preferred** over others?

Example: Between any two outcomes (a, b):

Preferences II

- We will allow **three possible answers**:

Preferences II

- We will allow **three possible answers**:

1. $a \succ b$: (Strictly) prefer a over b

Preferences II

- We will allow **three possible answers**:

1. $a \succ b$: (Strictly) prefer a over b
2. $a \prec b$: (Strictly) prefer b over a

Preferences II

- We will allow **three possible answers**:

1. $a \succ b$: (Strictly) prefer a over b
2. $a \prec b$: (Strictly) prefer b over a
3. $a \sim b$: Indifferent between a and b

Preferences II

- We will allow **three possible answers**:

1. $a \succ b$: (Strictly) prefer a over b
2. $a \prec b$: (Strictly) prefer b over a
3. $a \sim b$: Indifferent between a and b

- ***Preferences* are a list of all such comparisons between all bundles**

So What About the Numbers?

- Long ago (1890s), utility considered a real, measurable, **cardinal** scale†

- Utility thought to be lurking in people's brains

    - Could be understood from first principles: calories, water, warmth, etc

- Obvious problems

† "Neuroeconomics" & cognitive scientists are re-attempting a scientific approach to measure utility

Utility Functions?

- More plausibly **infer people's preferences from their actions**!
  - “Actions speak louder than words”
- **Principle of Revealed Preference**: if a person chooses x over y, and both are affordable, then they must prefer $x \succeq y$
- Flawless? Of course not. But extremely useful approximation!
  - People tend not to leave money on the table

Utility Functions!

- A **utility function** $u(\cdot)$[†] *represents* preference relations $(\succ, \prec, \sim)$

- Assign utility numbers to bundles, such that, for any bundles a and b:

$$a \succ b \iff u(a) > u(b)$$

[†] The $\cdot$ is a placeholder for whatever goods we are considering (e.g. x, y, burritos, lattes, dollars, etc)

Utility Functions, Pural I

Example: Imagine three alternative bundles of (x, y):

$$a = (1, 2)$$
$$b = (2, 2)$$
$$c = (4, 3)$$

- **Does this mean that bundle c is 3 times the utility of a?**

- Let $u(\cdot)$ assign each bundle a utility level:

$u(\cdot)$
$u(a) = 1$
$u(b) = 2$
$u(c) = 3$

Utility Functions, Pural II

Example: Imagine three alternative bundles of (x, y):

$$a = (1, 2)$$
$$b = (2, 2)$$
$$c = (4, 3)$$

- Now consider $u(\cdot)$ and a *2nd* function $v(\cdot)$:

$u(\cdot)$	$v(\cdot)$
$u(a) = 1$	$v(a) = 3$
$u(b) = 2$	$v(b) = 5$
$u(c) = 3$	$v(c) = 7$

Utility Functions, Pural III

- Utility numbers have an **ordinal** meaning only, **not cardinal**

- Both are valid utility functions:
  - $u(c) > u(b) > u(a)$ ✅
  - $v(c) > v(b) > v(a)$ ✅
  - because $c \succ b \succ a$

- **Only the <u>ranking</u> of utility numbers matters!**

Utility Functions and Payoffs Over Game Outcomes

- We want to apply utility functions to the outcomes in games, often summarized as **"payoff functions"**

- Using the **ordinal** interpretation of utility functions, we can rank player preferences over game outcomes

Utility Functions and Payoffs Over Game Outcomes

- Take a **prisoners' dilemma** and consider the payoffs to Player 1
- $u_1(D, C) \succ u_1(C, C)$
  - $0 > -6$
- $u_1(D, D) \succ u_1(C, D)$
  - $-12 > -24$

		Player 2: Cooperate	Player 2: Defect
Player 1	Cooperate	-6, -6	-24, 0
	Defect	0, -24	-12, -12

Utility Functions and Payoffs Over Game Outcomes

- Take a **prisoners' dilemma** and consider the payoffs to Player 2

- $u_2(C, D) \succ u_2(C, C)$
  - $0 > -6$
- $u_2(D, D) \succ u_2(D, C)$
  - $-12 > -24$

		Player 2: Cooperate	Player 2: Defect
Player 1	Cooperate	-6, -6	-24, 0
	Defect	0, -24	-12, -12

Utility Functions and Payoffs Over Game Outcomes

- We will keep the process simple for now by simply assigning numbers to consequences
- In fact, we can assign almost *any* numbers to the payoffs as long as we keep the *order* of the payoffs the same
  - i.e. so long as $u(a) > u(b)$ for all $a \succ b$

		Player 2: Cooperate	Player 2: Defect
Player 1	Cooperate	-6, -6	-24, 0
	Defect	0, -24	-12, -12

Utility Functions and Payoffs Over Game Outcomes

- We will keep the process simple for now by simply assigning numbers to consequences
- In fact, we can assign almost *any* numbers to the payoffs as long as we keep the *order* of the payoffs the same
  - i.e. so long as $u(a) > u(b)$ for all $a \succ b$

		Player 2: Cooperate	Player 2: Defect
Player 1	Cooperate	5, 5	7, 0
	Defect	0, 7	2, 2

This is the same game

Utility Functions and Payoffs Over Game Outcomes

- We will keep the process simple for now by simply assigning numbers to consequences
- In fact, we can assign almost *any* numbers to the payoffs as long as we keep the *order* of the payoffs the same
  - i.e. so long as $u(a) > u(b)$ for all $a \succ b$

Player 2

Player 1		Cooperate	Defect
	Cooperate	3, 3	1, 4
	Defect	4, 1	2, 2

This is the same game

Utility Functions and Payoffs Over Game Outcomes

- We will keep the process simple for now by simply assigning numbers to consequences
- In fact, we can assign almost *any* numbers to the payoffs as long as we keep the *order* of the payoffs the same
  - i.e. so long as $u(a) > u(b)$ for all $a \succ b$

		Player 2: Cooperate	Player 2: Defect
Player 1	Cooperate	b_1, b_2	d_1, a_2
	Defect	a_1, d_2	c_1, c_2

This is the same game, so long as

$$a > b > c > d$$

Rationality, Uncertainty, and Risk

- We commonly assume, for a game:
- Players understand the rules of the game
    - Common knowledge assumption
- Players behave **rationally**: try to maximize payoff
    - represented usually as (ordinal) utility
    - make no mistakes in choosing their strategies

Rationality, Uncertainty, and Risk

- Game theory does not permit us to consider true **uncertainty**
    - Must rule out *complete* surprises (Act of God, etc.)
    - What do people maximize in the presence of true uncertainty? Good question

- But we can talk about **risk**: distribution of outcomes occurring with some known **probability**

- In such cases, what do players **maximize** in the presence of risk?

Rationality, Uncertainty, and Risk

- One hypothesis: players choose strategy that maximizes **expected value** of payoffs
  - probability-weighted average
  - leads to a lot of paradoxes!

$$E[p] = \sum_{i=1}^{n} \pi_i p_i$$

- π is the probability associated with payoff p_i

Rationality, Uncertainty, and Risk

- Refinement by Von Neuman & Morgenstern: players instead maximize **expected *utility***
  - utility function over probabilistic outcomes
  - still some paradoxes, but fewer!

$$p_a \succ p_b \iff E[u(p_a)] > E[u(p_b)]$$

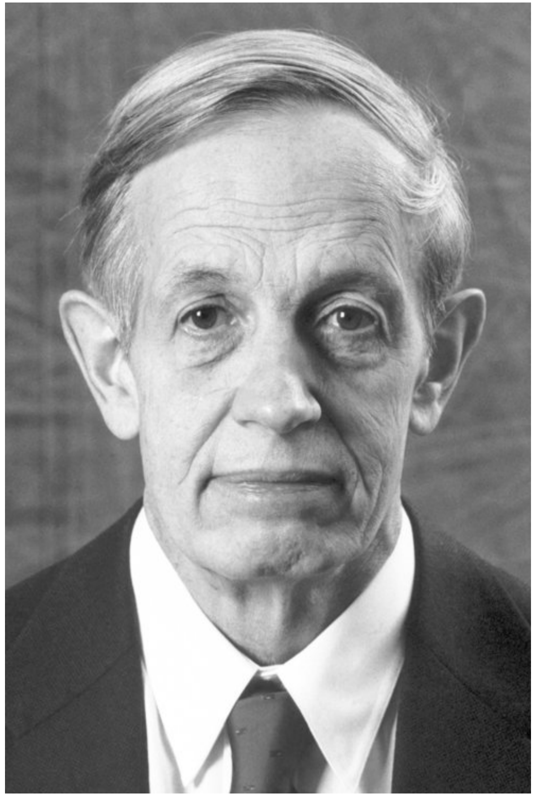
- Allows for different **risk attitudes**:
  - risk neutral, risk-averse, risk-loving
- makes utility functions **cardinal** (but still not measurable!)
  - called VNM utility functions

Solution Concepts: Nash Equilibrium

Advancing Game Theory

- Von Neumann & Morgenstern (vNM)'s *Theory of Games and Economic Behavior* (1944) establishes "Game theory"
- Solve for outcomes only of 2-player zero-sum games
- Minimax method (we'll see below)

Advancing Game Theory

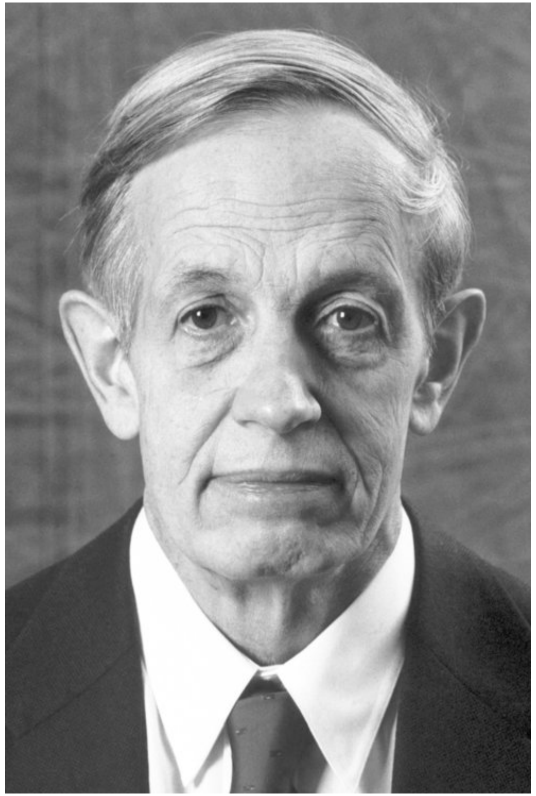


John Forbes Nash

1928—2015

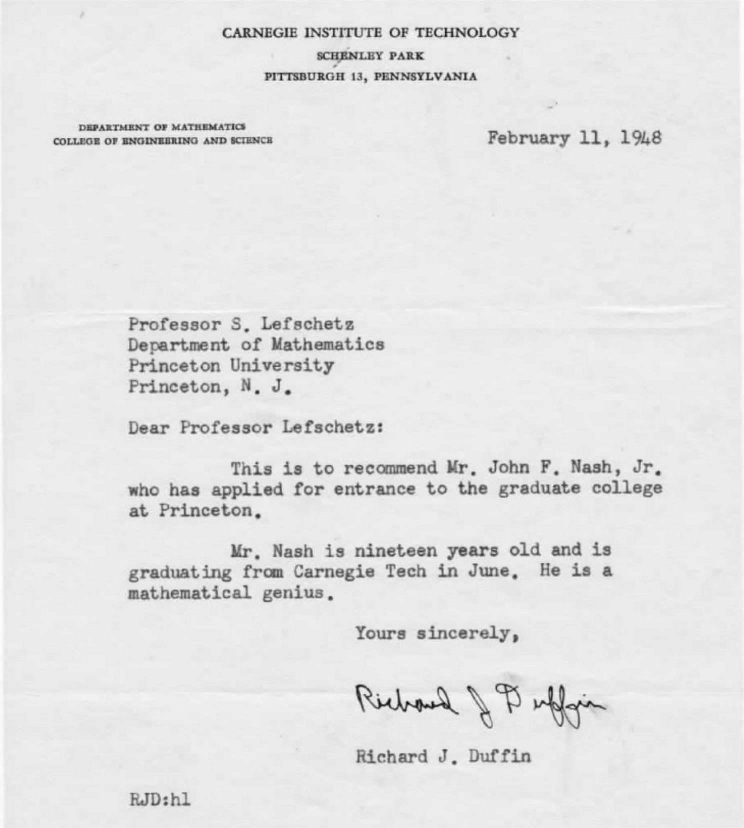
- Nash's *Non-Cooperative Games* (1950) dissertation invents idea of "(Nash) Equilibrium"
    - Extends for all n-player non-cooperative games (zero sum, negative sum, positive sum)
    - Proves an equilibrium exists for all games with finite number of players, strategies, and rounds
- Nash's 27 page Dissertation on Non-Cooperative Games

Advancing Game Theory



John Forbes Nash

1928—2015



CARNEGIE INSTITUTE OF TECHNOLOGY
SCHENLEY PARK
PITTSBURGH 13, PENNSYLVANIA

DEPARTMENT OF MATHEMATICS
COLLEGE OF ENGINEERING AND SCIENCE

February 11, 1948

Professor S. Lefschetz
Department of Mathematics
Princeton University
Princeton, N. J.

Dear Professor Lefschetz:

This is to recommend Mr. John F. Nash, Jr. who has applied for entrance to the graduate college at Princeton.

Mr. Nash is nineteen years old and is graduating from Carnegie Tech in June. He is a mathematical genius.

Yours sincerely,

Richard J. Duffin

Richard J. Duffin

RJD:hl

A Beautiful Movie, Lousy Economics

- A **Pure Strategy Nash Equilibrium (PSNE)** of a game is a set of strategies (one for each player) such that no player has a profitable deviation from their strategy given the strategies played by all other players

- Each player's strategy must be a best response to all other players' strategies

A Beautiful Movie, Lousy Economics

Solution Concepts: Nash Equilibrium

- Recall, **Nash Equilibrium**: no players want to change their strategy given what everyone else is playing
  - All players are playing a best response to each other

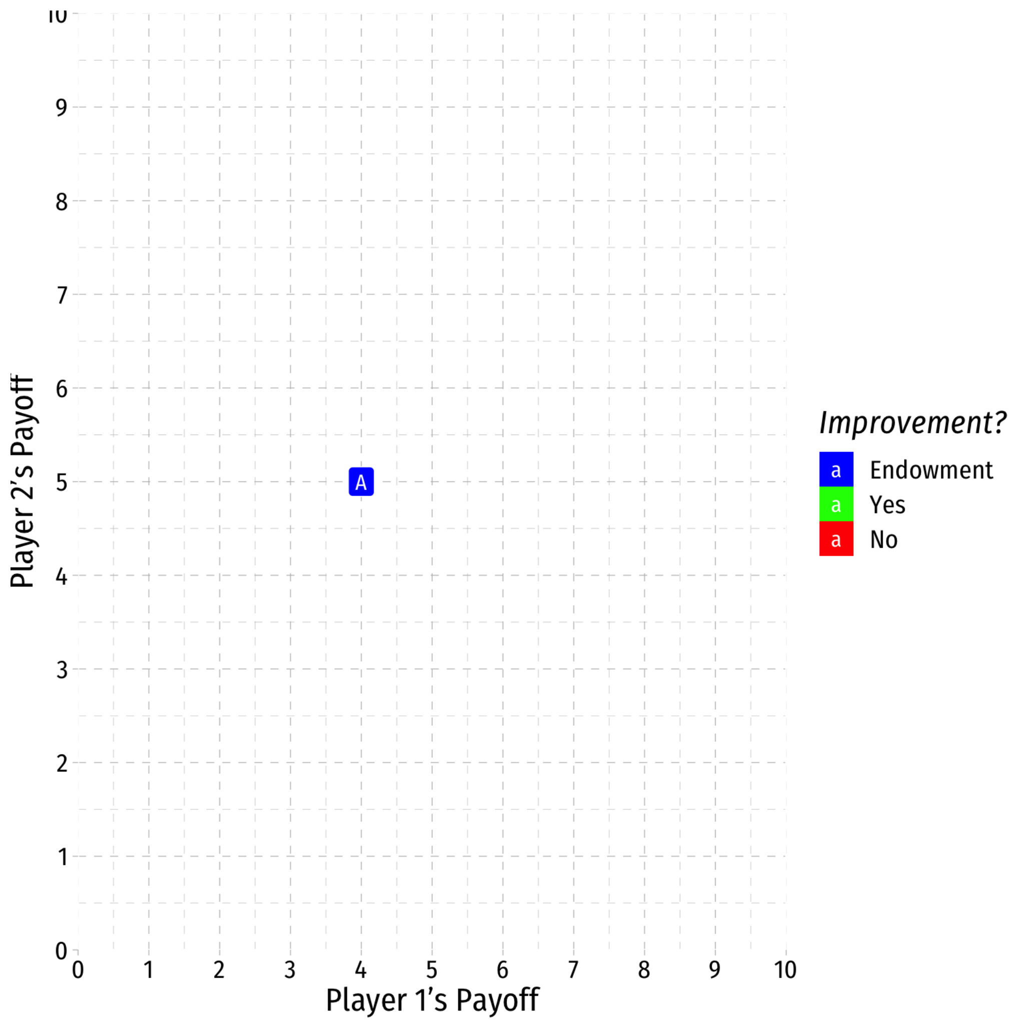
Solution Concepts: Nash Equilibrium

- Important about Nash equilibrium:

1. N.E. $\neq$ the "*best*" or *optimal* outcome
   - Recall the Prisoners' Dilemma!
2. Game may have *multiple* N.E.
3. Game may have *no* N.E. (in "pure" strategies)
4. All players are not necessarily playing the same strategy
5. Each player makes the same choice each time the game is played (possibility of mixed strategies)

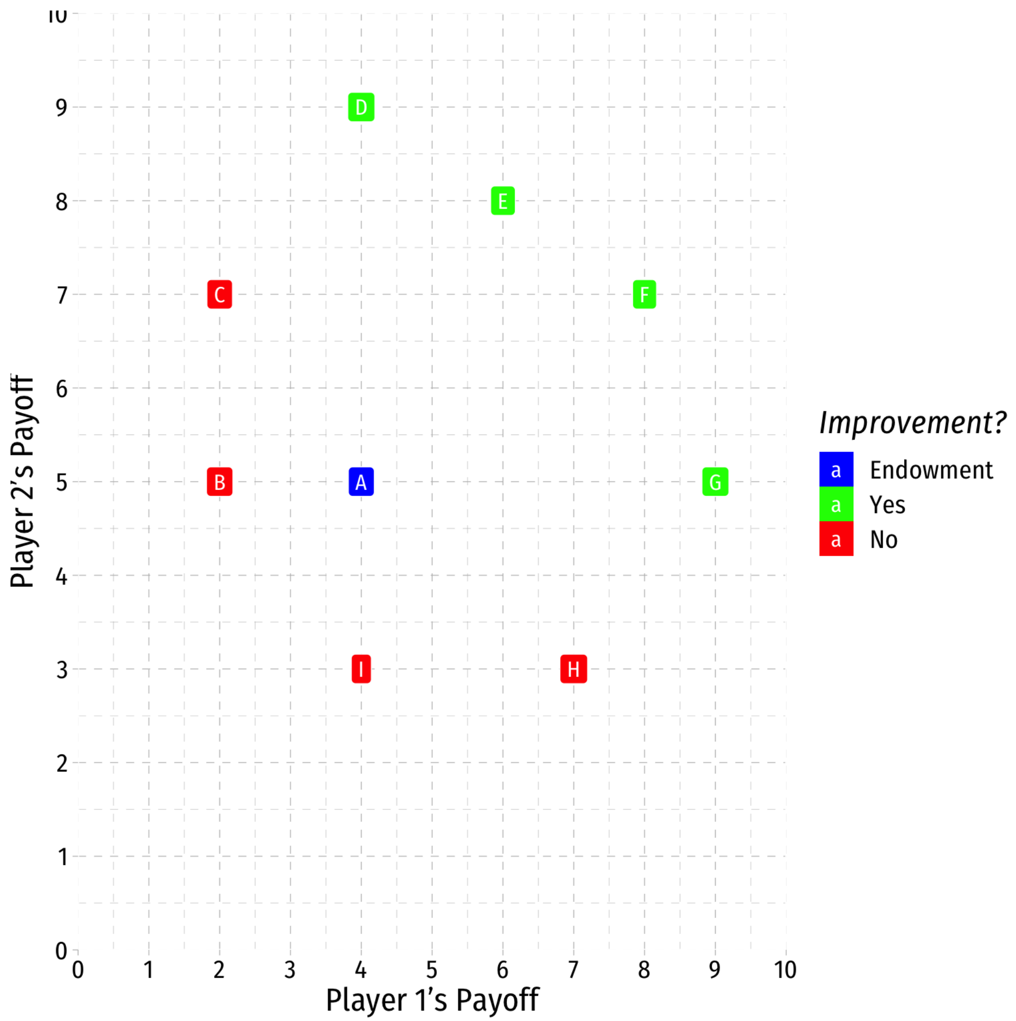
Pareto Efficiency

- Suppose we start from some initial allocation (A)



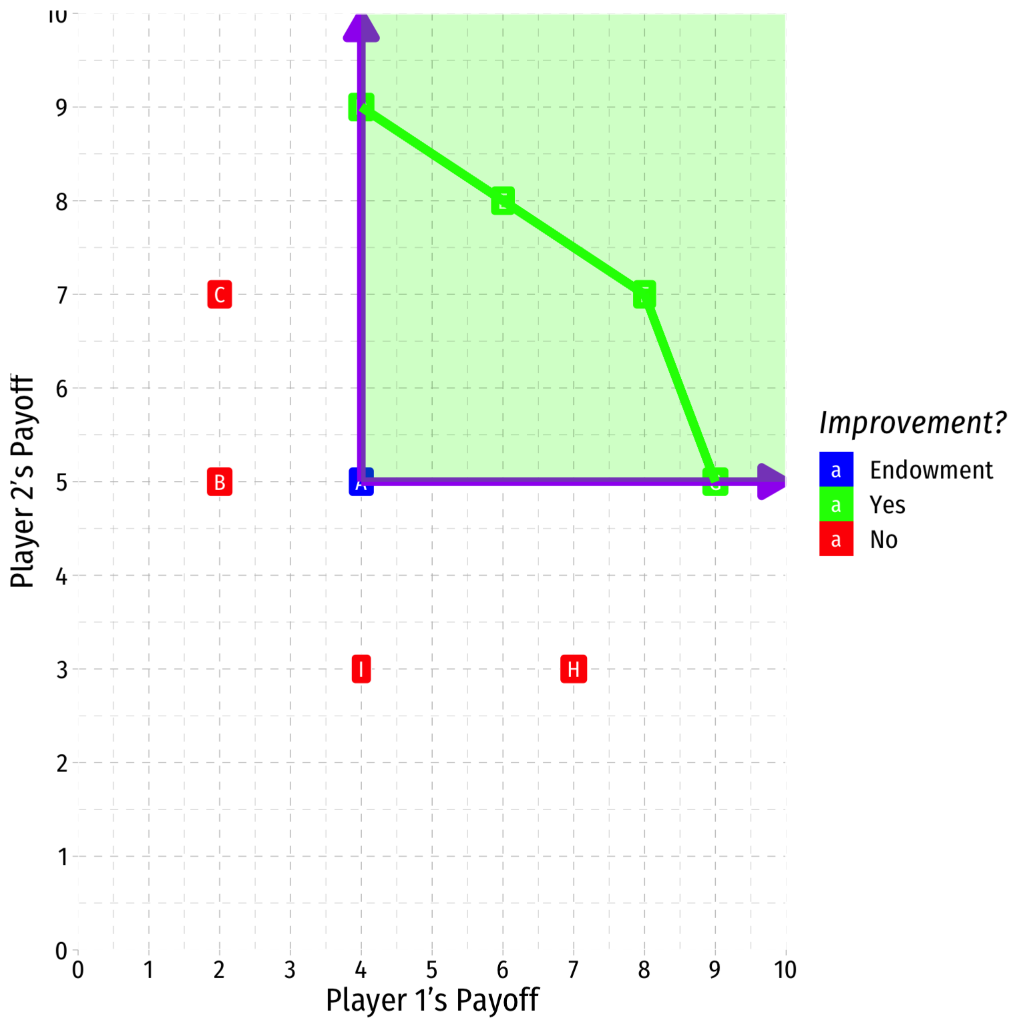
Pareto Efficiency

- Suppose we start from some initial allocation (A)
- **Pareto Improvement**: at least one party is better off, and no party is worse off
  - D, E, F, G are improvements
  - B, C, H, I are not



Pareto Efficiency

- Suppose we start from some initial allocation (A)
- **Pareto Improvement**: at least one party is better off, and no party is worse off
    - D, E, F, G are improvements
    - B, C, H, I are not
- **Pareto optimal/efficient**: no possible Pareto improvements
    - Set of Pareto efficient points often called the **Pareto frontier**†
    - Many possible efficient points!



†I'm simplifying...for full details, see class 1.8 appendix about applying consumer theory!

Pareto Efficiency and Games

- Take the **prisoners' dilemma**
- Nash Equilibrium: (Defect, Defect)
  - neither player has an incentive to change strategy, *given the other's strategy*
- Why can't they both **cooperate**?
  - A clear Pareto improvement!

Player 2

Player 1		Cooperate	Defect
	Cooperate	3, 3	1, 4
	Defect	4, 1	2, 2

Pareto Efficiency and Games

- Main feature of prisoners' dilemma: the Nash equilibrium is Pareto inferior to another outcome (**Cooperate**, **Cooperate**)!
    - But that outcome is *not* a Nash equilibrium!
    - Dominant strategies to **Defect**

- How can we ever get rational cooperation?

		Player 2: **Cooperate**	Player 2: **Defect**
Player 1	**Cooperate**	3, 3	1, <u>4</u>
	Defect	<u>4</u>, 1	<u>2</u>, <u>2</u>

Nash Equilibrium and Solution Concepts

- This is **far** from the last word on solution concepts, or even Nash equilibrium!

- But sufficient for now, until we return to simultaneous games

- Next week, **sequential games**!