Problem Set 4

ECON 316 — Game Theory — Fall 2021

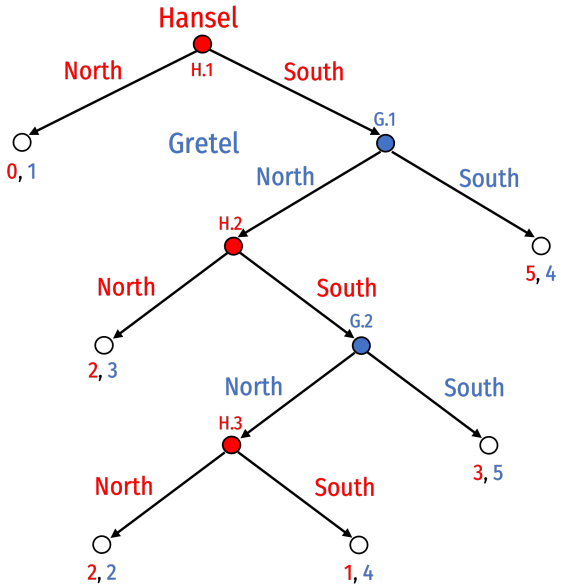
Due by Wednesday October 13

Concepts

1. What is the difference between a **finitely** repeated game and an **infinitely** repeated game? What is the pure strategy Nash equilibrium in a finitely-repeated game (with a unique Nash equilibrium in a one-shot version)? Describe *two* interpretations of *infinitely* repeated games.

2. Describe, in your own words, the simple version (or implications) of the **folk theorem** for sustaining cooperation.

3. Describe what a **subgame** means, and circle all subgames in the following game tree.



4. Define **subgame perfect Nash equilibrium**.

5. Explain what **strategic moves** are, and explain the three major types of strategic moves.

6. What makes a *promise* **credible**? What makes a *threat* **credible**? Give some examples of each, and in your answers, use the concept of **subgame perfection.**

7. What makes a strategy **evolutionarily stable** (ESS)? Describe the difference between **monomorphic** and **polymorphic** equilibria.

Problems

8. Consider an evolutionary version of the Stag Hunt game, where members of a species can decide to cooperate and hunt a **Stag** together, or defect and go after a **Hare** on their own.

		Player 2: Stag	Player 2: Hare
Player 1	Stag	2, 2	0, 1
	Hare	1, 0	1, 1

a. Is **Stag** an evolutionarily stable strategy (ESS)?
b. Is **Hare** an evolutionarily stable strategy (ESS)?
c. What are the pure strategy Nash equilibria (PSNE) of this game? Reconcile this with your answers in parts a and b.
d. Suppose the environment changes such that hunting a large **Hare** *alone* is equally rewarding to the cooperative hunt of a **Stag** (but if they both hunt **Hare**, it is less rewarding).

		Player 2: Stag	Player 2: Hare
Player 1	Stag	2, 2	0, 2
	Hare	2, 0	1, 1

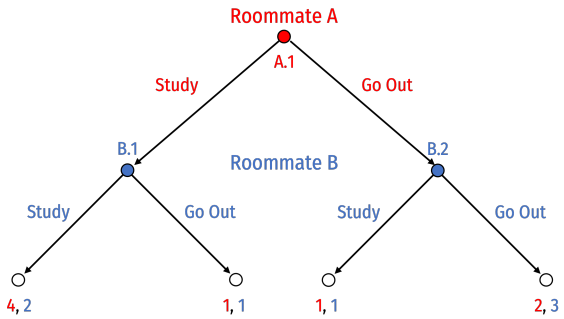
Under the new environment, is **Hare** evolutionarily stable (ESS)?
e. Under the new environment, is **Stag** evolutionarily stable (ESS)?
f. Given what we learned in class about the relationship between (pure strategy) Nash equilibria and evolutionarily stable strategies, we now need a new refinement. Define a ***strict* Nash equilibrium** in pure strategies to mean that each player is playing a **strict** (or **unique**) best response to other players, i.e. there is no *other* strategy that is *also* a best response to another player. In the one-shot game in part d, which PSNE are **strict**, and which are not (i.e. "weak" PSNE? What do you then think is the relationship between ESS and strict/non-strict PSNE?

9. Consider the evolutionary Hawk-Dove game, where members of a species are competing over a resource valued at 12, with a cost of losing a fight being -15.

a. Draw the payoff matrix for the game.
b. Find the pure strategy Nash equilibria.
c. Is **Hawk** evolutionarily stable?
d. Is **Dove** evolutionarily stable?
e. Reconcile your answers in parts c and d to your answer in part b.
f. Find the evolutionarily stable (polymorphic) equilibrium distribution of **Hawks** and **Doves.** [Hint: let p be the probability the *other* player is a **Hawk**.]

10. Consider the following game between two roommates. Roommate A has a very difficult exam the next morning, while Roommate B does not. The two of them can each decide to Study or Go Out that evening. Both would rather do something together, while A would certainly prefer they both Study and B would prefer they both Go Out.

a. Suppose they both agree that A gets to decide first and B must respond, as in the following game:



Solve this game for the rollback equilibrium using backwards induction.

b. Circle all subgames on the game tree.

c. Carefully convert this game from extensive form to strategic form. (Be mindful of how many potential strategies each player has!) Then, find any Nash equilibria in strategic form.

d. Which Nash equilibrium is subgame perfect? Why?

e. Suppose in an attempt to get A to Go Out, B says they will Go Out regardless of what A does. If A still gets to decide first (i.e. it is the same game as in part a), what should A make of this?