1.4 — Simultaneous Games & Normal For

ECON 316 • Game Theory • Fall 2021

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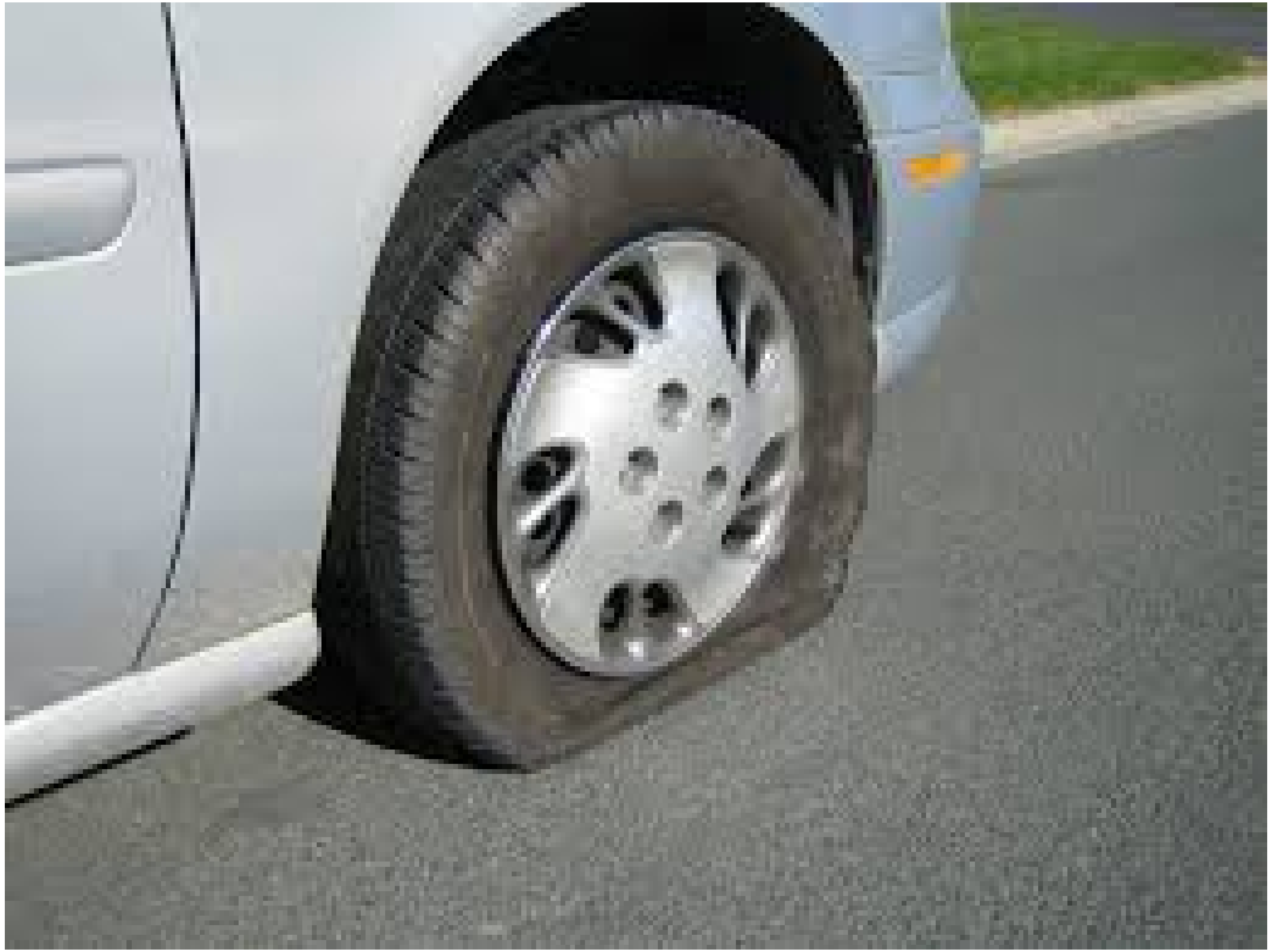
Outline

Simultaneous Games

Simultaneous Games

- Players must make choices simultaneously, but under **strategic uncertainty**
    - Don't know which strategies other players are playing before you choose yours
- *Possible* strategic choices and payoffs of each outcome to each player *are* known by all players
- Must think not only about own best strategic choice, but also the best strategic choice of *other* player(s)

Flat Tire Story



Games in Normal Form

- Normal or strategic form
- By convention **Row Player** is Player 1, **Column player** is Player 2
  - First payoff in a cell goes to **Row**, second to **Column**
  - But order doesn't matter (!)
- Dimensions of matrix
  - Rows: possible strategies available to **Row**
  - Columns: possible strategies available to **Column**
- For now, we only look at **discrete** strategies (and a single decision per player)

		Friend			
		Front L	**Front R**	**Rear L**	**Rear R**
You	**Front L**	1, 1	0, 0	0, 0	0, 0
	Front R	0, 0	1, 1	0, 0	0, 0
	Rear L	0, 0	0, 0	1, 1	0, 0
	Rear R	0, 0	0, 0	0, 0	1, 1

Nash Equilibrium, Again

- Again, in a **Nash equilibrium**, no player wants to change strategies given the strategies played by all other players
  - Equivalently, each player is playing a best response to other players' strategies
- Today we will learn **several methods** to search for Nash equilibria in simultaneous games

Cell-by-Cell Inspection

- Consider again the **prisoners' dilemma**

- Consider each outcome and ask, **does *any* player want to change strategies, *given what the other player is doing?***

  1. (C, C)
  2. (C, D)
  3. (D, C)
  4. (D, D)

		Player 2: Cooperate	Player 2: Defect
Player 1	Cooperate	3, 3	1, 4
Player 1	Defect	4, 1	2, 2

Cell-by-Cell Inspection

- Consider again the prisoners' dilemma
- Consider each outcome and ask, **does *any* player want to change strategies, *given what the other player is doing*?**

  1. (C, C) ✅
  2. (C, D) ✅
  3. (D, C) ✅
  4. (D, D) ❌

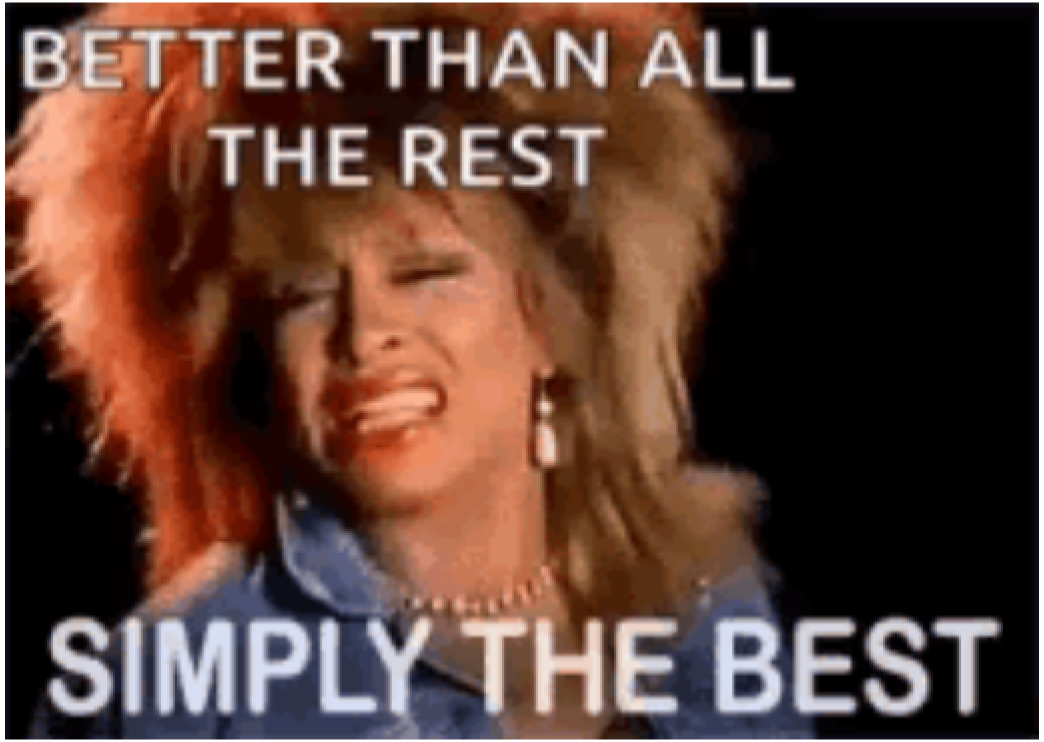
- If no player wants to switch strategies (given the others’), that outcome is a Nash equilibrium: (D, D)

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	3, 3	1, 4
	Defect	4, 1	2, 2

Dominance Solvability

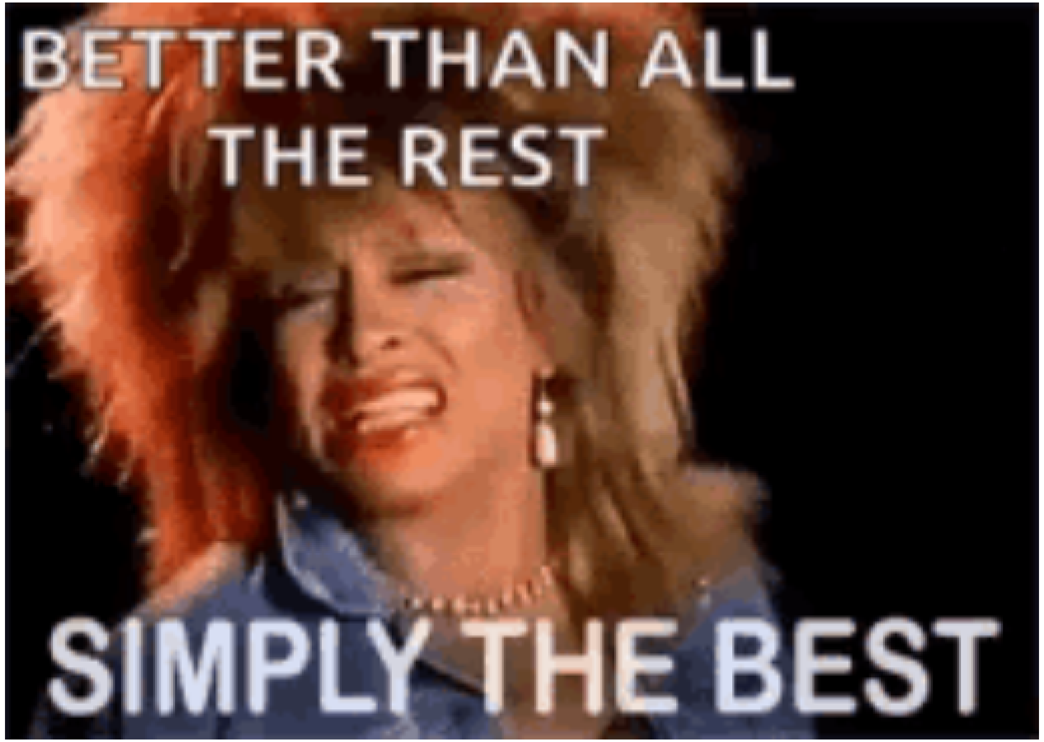
Dominance Solvability

- One efficient (but not foolproof) method for finding solution: search for **dominated strategies** and eliminate them
  - like pruning branches of a sequential game tree



Dominance Solvability

- A player has a **dominant strategy** when it yields a *higher* payoff than *all other* strategies available, regardless of what strategy the other player is playing
- A player has a **dominated strategy** when it yields a *lower* payoff than *all other* strategies available, regardless of what strategy the other player is playing



Dominance Solvability

- Consider the **prisoners' dilemma**

		Player 2: Cooperate	Player 2: Defect
Player 1	Cooperate	3, 3	1, 4
	Defect	4, 1	2, 2

Dominance Solvability

- Consider the **prisoners' dilemma**
- For **Player 1**...

		Player 2	
		Cooperate	**Defect**
Player 1	**Cooperate**	3 3	1 4
	Defect	4 1	2 2

Dominance Solvability

- Consider the **prisoners' dilemma**
- For **Player 1**...

			Player 2	
			Cooperate	Defect
Player 1	Cooperate		3, 3	1, 4
	Defect		4, 1	2, 2

Dominance Solvability

- Consider the **prisoners' dilemma**
- For **Player 1**...

		Player 2	
		Cooperate	**Defect**
Player 1	**Cooperate**	3, 3	1, 4
	Defect	4, 1	2, 2

Dominance Solvability

- Consider the **prisoners' dilemma**
- For **Player 1**...

		Player 2: Cooperate	Player 2: Defect
Player 1	Cooperate	3, 3	1, 4
	Defect	4, 1	2, 2

Dominance Solvability

- Consider the **prisoners' dilemma**
- For **Player 1**: Cooperate is **dominated** by Defect
  - $u_1(\color{red}{D}, \color{blue}{C}) \succ u_1(\color{red}{C}, \color{blue}{C})$
  - $u_1(\color{red}{D}, \color{blue}{D}) \succ u_1(\color{red}{C}, \color{blue}{D})$

		Player 2	
		Cooperate	**Defect**
Player 1	**Cooperate**	3, 3	1, 4
	Defect	4, 1	2, 2

Dominance Solvability

- Consider the **prisoners' dilemma**
- For **Player 1**: Cooperate is **dominated** by Defect
  - $u_1(\color{red}{D}, \color{blue}{C}) \succ u_1(\color{red}{C}, \color{blue}{C})$
  - $u_1(\color{red}{D}, \color{blue}{D}) \succ u_1(\color{red}{C}, \color{blue}{D})$
- Knowing **Player 1** will **never** play Cooperate, we can delete that entire row

		Player 2	
		Cooperate	**Defect**
Player 1	**Defect**	4 1	2 2

Dominance Solvability

- Consider the **prisoners' dilemma**
- For **Player 1**: Cooperate is **dominated** by Defect
  - $u_1(\color{red}{D}, \color{blue}{C}) \succ u_1(\color{red}{C}, \color{blue}{C})$
  - $u_1(\color{red}{D}, \color{blue}{D}) \succ u_1(\color{red}{C}, \color{blue}{D})$
- Knowing **Player 1** will **never** play Cooperate, we can delete that entire row

		Player 2	
		Cooperate	**Defect**
Player 1	**Defect**	4 1	2 2

Dominance Solvability

- Alternatively, we could consider **Player 2**

		Player 2: Cooperate	Player 2: Defect
Player 1	Cooperate	3, 3	1, 4
	Defect	4, 1	2, 2

Dominance Solvability

- Alternatively, we could consider **Player 2**
- For **Player 2**...

		Player 2	
		Cooperate	**Defect**
Player 1	**Cooperate**	3, 3	1, 4
	Defect	4, 1	2, 2

Dominance Solvability

- Alternatively, we could consider **Player 2**
- For **Player 2**...

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	3, 3	1, 4
	Defect	4, 1	2, 2

Dominance Solvability

- Alternatively, we could consider **Player 2**
- For **Player 2**...

		Player 2: Cooperate	Player 2: Defect
Player 1	Cooperate	3, 3	1, 4
Player 1	Defect	4, 1	2, 2

Dominance Solvability

- Alternatively, we could consider **Player 2**
- For **Player 2**…

		Player 2 Cooperate	Player 2 Defect
Player 1	Cooperate	3, 3	1, 4
Player 1	Defect	4, 1	2, 2

Dominance Solvability

- Alternatively, we could consider **Player 2**
- For **Player 2**: Cooperate is **dominated** by Defect
  - $u_2(\color{red}{C}, \color{blue}{D}) \succ u_2(\color{red}{C}, \color{blue}{C})$
  - $u_2(\color{red}{D}, \color{blue}{D}) \succ u_2(\color{red}{D}, \color{blue}{C})$

		Player 2	
		Cooperate	**Defect**
Player 1	**Cooperate**	3, 3	1, 4
	Defect	4, 1	2, 2

Dominance Solvability

- Alternatively, we could consider **Player 2**
- For **Player 2**: Cooperate is **dominated** by Defect
  - $u_2(\color{red}{C}, \color{blue}{D}) \succ u_2(\color{red}{C}, \color{blue}{C})$
  - $u_2(\color{red}{D}, \color{blue}{D}) \succ u_2(\color{red}{D}, \color{blue}{C})$
- Knowing **Player 2** will **never** play Cooperate, we can delete that entire

		Player 2
		Defect
Player 1	**Cooperate**	1, 4
	Defect	2, 2

Dominance Solvability

- Alternatively, we could consider **Player 2**
- For **Player 2**: Cooperate is **dominated** by Defect
  - $u_2(\color{red}{C}, \color{blue}{D}) \succ u_2(\color{red}{C}, \color{blue}{C})$
  - $u_2(\color{red}{D}, \color{blue}{D}) \succ u_2(\color{red}{D}, \color{blue}{C})$
- Knowing **Player 2** will **never** play Cooperate, we can delete that entire

Player 2

		Defect
Player 1	**Cooperate**	1, 4
	Defect	2, 2

Dominance Solvability

- Take the **prisoners' dilemma**
- Nash Equilibrium: (Defect, Defect)
  - neither player has an incentive to change strategy, *given the other's strategy*
- Why can't they both **cooperate**?
  - A clear Pareto improvement!

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	3, 3	1, 4
	Defect	4, 1	2, 2

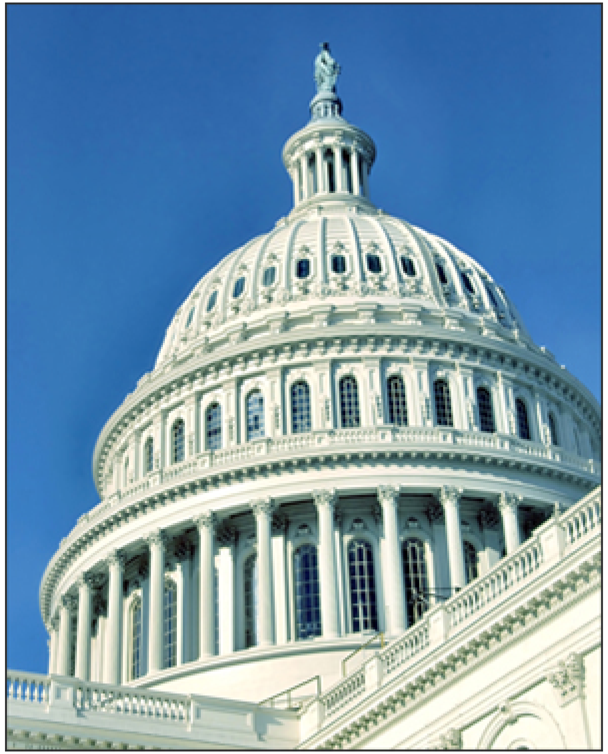
Pareto Efficiency and Games

- Main feature of prisoners' dilemma: the Nash equilibrium is Pareto inferior to another outcome (**Cooperate**, **Cooperate**)!
  - But that outcome is *not* a Nash equilibrium!
  - Dominant strategies to **Defect**
- How can we ever get rational cooperation?

		Player 2	
		Cooperate	**Defect**
Player 1	**Cooperate**	3, 3	1, <u>4</u>
	Defect	<u>4</u>, 1	<u>2</u>, <u>2</u>

When One Player Has a Dominant Strategy

- Congress determines fiscal policy
- Can tax & spend to Balance Budget
- Can tax & spend to run a Budget Deficit
- Constant political pressure to spend more & tax less
    - May raise possibility of inflation



When One Player Has a Dominant Strategy

- **Federal Reserve** determines monetary policy
- Can target Low Interest Rates
- Can target High Interest Rates
- Generally wants to avoid inflation
  - Likes keeping interest rates low to stimulate Demand (if no threat of inflation)

When One Player Has a Dominant Strategy

- Both players choose policy simultaneously and independently of each other

- How to find the equilibrium of this game?

		Federal Reserve	
		Low Rates	High Rates
Congress	Balance Budget	3, 4	1, 3
	Budget Deficit	4, 1	2, 2

When One Player Has a Dominant Strategy

- Both players choose policy simultaneously and independently of each other
- How to find the equilibrium of this game?
    - Does the **Fed** have a dominant strategy?

		Federal Reserve: Low Rates	Federal Reserve: High Rates
Congress	Balance Budget	3, 4	1, 3
Congress	Budget Deficit	4, 1	2, 2

When One Player Has a Dominant Strategy

- Both players choose policy simultaneously and independently of each other
- How to find the equilibrium of this game?
    - Does the **Fed** have a dominant strategy?
    - Does **Congress**?

		Federal Reserve	
		Low Rates	**High Rates**
Congress	**Balance Budget**	3 4	1 3
	Budget Deficit	4 1	2 2

When One Player Has a Dominant Strategy

- Both players choose policy simultaneously and independently of each other

- How to find the equilibrium of this game?
  - Does the **Fed** have a dominant strategy?
  - Does **Congress**?
  - Given this, how will **Fed** choose?

		Federal Reserve	
		Low Rates	**High Rates**
Congress	**Budget Deficit**	4, 1	2, 2

Successive Elimination of Dominated Strategies

- What about the following game?

Row		Column: Left	Column: Middle	Column: Right
	Up	3, 1	2, 3	10, 2
	Down	4, 5	3, 0	6, 4
	Left	2, 2	5, 4	12, 3
	Right	5, 6	4, 5	9, 7

Successive Elimination of Dominated Strategies

- What about the following game?
- Hint: Do any of **Row**'s strategies *always* yield a lower payoff than another strategy?

Row \ Column	Left	Middle	Right
Up	3, 1	2, 3	10, 2
Down	4, 5	3, 0	6, 4
Left	2, 2	5, 4	12, 3
Right	5, 6	4, 5	9, 7

Successive Elimination of Dominated Strategies

- What about the following game?
- Hint: Do any of **Row**'s strategies *always* yield a lower payoff than another strategy?
  - Down is dominated by Right

		Column		
		Left	Middle	Right
Row	Up	3, 1	2, 3	10, 2
	Down	4, 5	3, 0	6, 4
	Left	2, 2	5, 4	12, 3
	Right	5, 6	4, 5	9, 7

Successive Elimination of Dominated Strategies

- What about the following game?
- Hint: Do any of **Row**'s strategies *always* yield a lower payoff than another strategy?
  - Down is dominated by Right
  - Remove this row, since **Row** will never play Down

		Column		
		Left	**Middle**	**Right**
Row	**Up**	3 1	2 3	10 2
	Left	2 2	5 4	12 3
	Right	5 6	4 5	9 7

Successive Elimination of Dominated Strategies

- Keep searching for dominated strategies...
- Hint: Do any of **Column**'s strategies *always* yield a lower payoff than another strategy?

		Column		
		Left	Middle	Right
Row	Up	3, 1	2, 3	10, 2
	Left	2, 2	5, 4	12, 3
	Right	5, 6	4, 5	9, 7

Successive Elimination of Dominated Strategies

- Keep searching for dominated strategies...
- Hint: Do any of **Column**'s strategies *always* yield a lower payoff than another strategy?
  - Left is dominated by Right
  - Remove this column, since **Column** will never play Left

Column

Row		**Left**	**Middle**	**Right**
	Up	3 1	2 3	10 2
	Left	2 2	5 4	12 3
	Right	5 6	4 5	9 7

Successive Elimination of Dominated Strategies

- Keep searching for dominated strategies…

		Column: Middle	Column: Right
Row	Up	2, 3	10, 2
	Left	5, 4	12, 3
	Right	4, 5	9, 7

Successive Elimination of Dominated Strategies

- Keep searching for dominated strategies…
- For **Row**, Left dominates *both* Up and Right
  - Delete both Up and Right since **Row** will never play them

		Column	
		Middle	**Right**
	Up	2, 3	10, 2
Row	**Left**	5, 4	12, 3
	Right	4, 5	9, 7

Successive Elimination of Dominated Strategies

- Keep searching for dominated strategies...
- Since **Row** will play Left, **Column**'s best response is to play Middle

		Column	
		Middle	**Right**
Row	**Left**	5, 4	12, 3

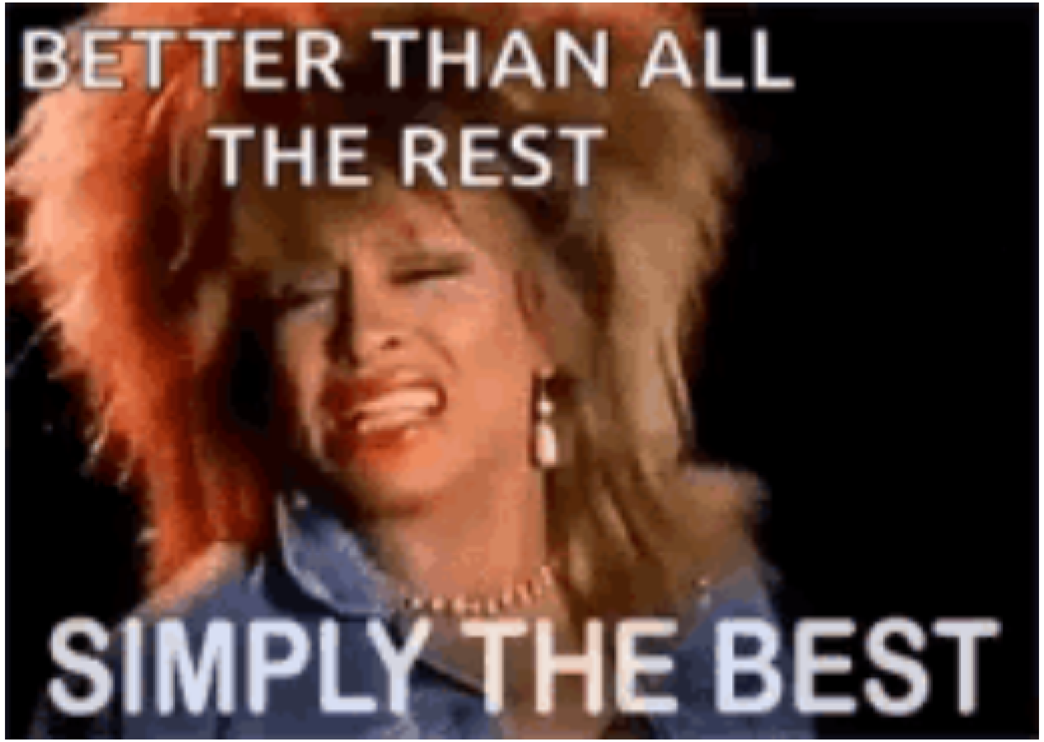
Successive Elimination of Dominated Strategies

- We've found the **Nash Equilibrium**: (Left, Middle)
- Check that it's truly an equilibrium
  - Does **Row** want to change from Left, given **Column** is playing Middle?
  - Does **Column** want to change from Middle, given **Row** is playing Left?

		Column		
		Left	**Middle**	**Right**
Row	**Up**	3, 1	2, 3	10, 2
	Down	4, 5	3, 0	6, 4
	Left	2, 2	5, 4	12, 3
	Right	5, 6	4, 5	9, 7

Successive Elimination of Dominated Strategies

- If **successive elimination of dominated strategies** yields a unique outcome, then the game is **"dominance solvable"**
  - Not all games can be solved this way!



You Try

		NBC		
		Sitcom	Talent	Game
CBS	Sitcom	55, 45	52, 48	51, 49
	Talent	50, 50	45, 55	46, 54
	Game	52, 48	49, 51	48, 52

Eliminating Dominated Strategies: Not Foolproof

- What about ties?

		Column	
		A	B
Row	A	0, 0	1, 1
	B	1, 1	1, 1

Eliminating Dominated Strategies: Not Foolproof

- What about ties?
- For **Row**, A is **“weakly” dominated** by B
  - If **Column** plays A, then playing B is strictly better than A for **Row**
  - If **Column** plays B, then playing B is at least as good $(\succsim)$ as A for **Row**

Column

Row	**A**	**B**
A	0, 0	1, 1
B	1, 1	1, 1

Eliminating Dominated Strategies: Not Foolproof

- What about ties?
- Same for **Column**: A is **"weakly" dominated** by B
  - If **Row** plays A, then playing B is strictly better than A for **Column**
  - If **Row** plays B, then playing B is at least as good \((\succsim)\) as A for **Column**

Column

Row	A	B
A	0, 0	1, 1
B	1, 1	1, 1

Eliminating Dominated Strategies: Not Foolproof

- Successive elimination of ***weakly*** dominated strategies implies deleting **A** for both players

- Predicted **Nash Equilibrium**: (B, B)

		Column A	Column B
Row	**A**	0, 0	1, 1
	B	1, 1	1, 1

Eliminating Dominated Strategies: Not Foolproof

- Successive elimination of ***weakly*** dominated strategies implies deleting **A** for both players

- Predicted **Nash Equilibrium**: (B, B)

		B
Row	B	1, 1

Eliminating Dominated Strategies: Not Foolproof

- Successive elimination of ***weakly*** dominated strategies implies deleting **A** for both players
- Predicted **Nash Equilibrium**: (B, B)
- But (A, B) and (B, A) are **also** Nash equilibria!
    - Check for yourself
- **So we can only rule out *strictly* dominated strategies!**

		Column	
		A	**B**
Row	**A**	0, 0	1, 1
	B	1, 1	1, 1

Best Response Analysis

Best Response Analysis

- Consider this game again, and check for each player's **best response** to each of the other player's strategies

		Column		
		Left	Middle	Right
Row	Up	3 1	2 3	10 2
	Down	4 5	3 0	6 4
	Left	2 2	5 4	12 3
	Right	5 6	4 5	9 7

Best Response Analysis

- Consider **Row**
  - If **Column** plays Left

		Column		
		Left	Middle	Right
Row	Up	3, 1	2, 3	10, 2
	Down	4, 5	3, 0	6, 4
	Left	2, 2	5, 4	12, 3
	Right	5, 6	4, 5	9, 7

Best Response Analysis

- Consider **Row**
  - If **Column** plays Left, best response is Right

Column

Row

	Left	**Middle**	**Right**
Up	3 1	2 3	10 2
Down	4 5	3 0	6 4
Left	2 2	5 4	12 3
Right	5 6	4 5	9 7

Best Response Analysis

- Consider **Row**
  - If **Column** plays Left, best response is Right
  - If **Column** plays Middle, best response is Left

		Column		
		Left	Middle	Right
Row	Up	3, 1	2, 3	10, 2
	Down	4, 5	3, 0	6, 4
	Left	2, 2	5, 4	12, 3
	Right	5, 6	4, 5	9, 7

Best Response Analysis

- Consider **Row**
  - If **Column** plays Left, best response is Right
  - If **Column** plays Middle, best response is Left
  - If **Column** plays Right, best response is Left

Column

Row		**Left**	**Middle**	**Right**
	Up	3, 1	2, 3	10, 2
	Down	4, 5	3, 0	6, 4
	Left	2, 2	<u>5</u>, 4	<u>12</u>, 3
	Right	<u>5</u>, 6	4, 5	9, 7

Best Response Analysis

- Consider **Column**
  - If **Row** plays Up

Column

Row		Left	Middle	Right
	Up	3 1	2 3	10 2
	Down	4 5	3 0	6 4
	Left	2 2	5 4	12 3
	Right	5 6	4 5	9 7

Best Response Analysis

- Consider **Column**
  - If **Row** plays Up, best response is Middle

Column

Row	Left	Middle	Right
Up	3, 1	2, 3	10, 2
Down	4, 5	3, 0	6, 4
Left	2, 2	5, 4	12, 3
Right	5, 6	4, 5	9, 7

Best Response Analysis

- Consider **Column**
  - If **Row** plays Up, best response is Middle
  - If **Row** plays Down, best response is Left

		Column: Left	Column: Middle	Column: Right
Row	Up	3, 1	2, 3	10, 2
Row	Down	4, 5	3, 0	6, 4
Row	Left	2, 2	5, 4	12, 3
Row	Right	5, 6	4, 5	9, 7

Best Response Analysis

- Consider **Column**
  - If **Row** plays Up, best response is Middle
  - If **Row** plays Down, best response is Left
  - If **Row** plays Left, best response is Middle

		Column		
		Left	**Middle**	**Right**
	Up	3, 1	2, 3	10, 2
	Down	4, 5	3, 0	6, 4
Row	**Left**	2, 2	5, 4	12, 3
	Right	5, 6	4, 5	9, 7

Best Response Analysis

- Consider **Column**
    - If **Row** plays Up, best response is Middle
    - If **Row** plays Down, best response is Left
    - If **Row** plays Left, best response is Middle
    - If **Row** plays Right, best response is Right

		Column		
		Left	**Middle**	**Right**
Row	**Up**	3, 1	2, 3	10, 2
	Down	4, 5	3, 0	6, 4
	Left	2, 2	5, 4	12, 3
	Right	5, 6	4, 5	9, 7

Best Response Analysis

- Highlighted all best responses for each player, shows us the **Nash Equilibrium**: (Left, Middle)
- In a Nash equilibrium, **all players are playing a best response to each other's strategies**
- A more tedious process, but foolproof

Row \ Column	Left	Middle	Right
Up	3, 1	2, 3	10, 2
Down	4, 5	3, 0	6, 4
Left	2, 2	5, 4	12, 3
Right	5, 6	4, 5	9, 7

Best Response Analysis Permits Ties

- For **Row** in this game:
  - If **Column** plays A, **Row**'s best response is B
  - If **Column** plays B, A and B are both best responses
- Symmetrically for **Column**
- Finds all three Nash equilibria (in each, both players play a best response)
  1. (B, A)
  2. (A, B)
  3. (B, B)

		Column A	Column B
Row	A	0, 0	1, 1
	B	1, 1	1, 1

Depicting Three Player Games

Depicting Three Player Games

ABC chooses Sitcom

		NBC: Sitcom	NBC: Game Show
CBS	Sitcom	34 25 41	32 32 36
	Game Show	32 30 38	33 31 36

ABC chooses Game Show

		NBC: Sitcom	NBC: Game Show
CBS	Sitcom	34 29 37	38 32 30
	Game Show	35 38 27	36 39 25

- Represent **ABC**'s choice across two matrices
- Three payoffs for each outcome: (CBS, NBC, ABC)
- Let's first try solving by searching for dominated strategies...
- Game Show is dominated by Sitcom for **ABC**, so delete it

Depicting Three Player Games

ABC chooses Sitcom

		NBC	
		Sitcom	Game Show
CBS	Sitcom	34 25 41	32 32 36
	Game Show	32 30 38	33 31 36

- Keep searching
- Sitcom is dominated by Game Show for **NBC**, so delete it

Depicting Three Player Games

ABC chooses **Sitcom**

		Game Show
CBS	**Sitcom**	32 32 36
	Game Show	33 31 36

- Keep searching
- **Sitcom** is dominated by Game Show for **CBS**, so delete it

Depicting Three Player Games

ABC chooses Sitcom

		Game Show
CBS	Game Show	33 31 36

- **Nash Equilibrium**: (Game Show, Game Show, Sitcom)

Depicting Three Player Games

ABC chooses Sitcom

		NBC: Sitcom	NBC: Game Show
CBS	Sitcom	34 25 41	32 32 36
	Game Show	32 30 38	33 31 36

ABC chooses Game Show

		NBC: Sitcom	NBC: Game Show
CBS	Sitcom	34 29 37	38 32 30
	Game Show	35 38 27	36 39 25

- **Nash Equilibrium**: (Game Show, Game Show, Sitcom)
- Now let's try using best response analysis instead

Depicting Three Player Games

ABC chooses Sitcom

		NBC: Sitcom	NBC: Game Show
CBS	Sitcom	34 25 41	32 32 36
	Game Show	32 30 38	33 31 36

ABC chooses Game Show

		NBC: Sitcom	NBC: Game Show
CBS	Sitcom	34 29 37	38 32 30
	Game Show	35 38 27	36 39 25

- Start with **CBS**
  - If **NBC** chooses Sitcom and **ABC** chooses Sitcom

Depicting Three Player Games

ABC chooses Sitcom

		NBC: Sitcom	NBC: Game Show
CBS	Sitcom	<u>34</u> 25 41	32 32 36
CBS	Game Show	32 30 38	33 31 36

ABC chooses Game Show

		NBC: Sitcom	NBC: Game Show
CBS	Sitcom	34 29 37	38 32 30
CBS	Game Show	35 38 27	36 39 25

- Start with **CBS**
  - If **NBC** chooses Sitcom and **ABC** chooses Sitcom, **CBS**' BR: Sitcom

Depicting Three Player Games

ABC chooses Sitcom

CBS \ NBC	Sitcom	Game Show
Sitcom	34 25 41	32 32 36
Game Show	32 30 38	33 31 36

ABC chooses Game Show

CBS \ NBC	Sitcom	Game Show
Sitcom	34 29 37	38 32 30
Game Show	35 38 27	36 39 25

- Start with **CBS**
    - If **NBC** chooses Sitcom and **ABC** chooses Sitcom, **CBS**' BR: Sitcom
    - If **NBC** chooses Game Show and **ABC** chooses Sitcom, **CBS**' BR: Game Show

Depicting Three Player Games

ABC chooses Sitcom

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Depicting Three Player Games

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- Now consider **NBC**

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- **Nash Equilibrium**: (Game Show, Game Show, Sitcom)

Summary of Methods of Finding Nash Eq.

Ranked from (most to least) effective and (most to least) tedious:

1. **Cell-by-cell inspection**
    - For each outcome, ask: would any player like to change strategy given others' strategies?
    - Every outcome where all players answer “NO” is a Nash equilibrium
2. **Best response analysis**
    - For each possible strategy of *other* players, what is a player's best response?
    - If **all** players are playing a best response in an outcome, that's a Nash equilibrium
3. **Successive elimination of dominated strategies**
    - Eliminate (dominated) strategies players will never play
    - If a single strategy remains for each player, that's the Nash equilibrium
    - Ties cause you to rule out potential Nash equilibria!