3.1 — Mixed Strategies

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Outline

When Pure Strategies Won't Work

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Oskar Morgenstern

1902—1977

“Sherlock Holmes, pursued by his opponent, Moriarty, leaves London for Dover. The train stops at a station on the way, and he alights there rather than travelling on to Dover. He has seen Moriarty at the railway station, recognizes that he is very clever and expects that Moriarty will take a faster special train in order to catch him in Dover. Holmes’s anticipation turns out to be correct. But what if Moriarty had been still more clever, had estimated Holmes’s mental abilities better and had foreseen his actions accordingly? Then, obviously, he would have travelled to the intermediate station [Canterbury]. Holmes again would have had to calculate that, and he himself would have decided to go on to Dover. Whereupon, Moriarty would again have ‘reacted’ differently,” (p.173-4).

When Pure Strategies Won't Work

> "'All that I have to say has already crossed your mind,' said he. 'Then possibly my answer has crossed yours,' I replied. 'You stand fast?' 'Absolutely.'"

— Arthur Conan Doyle, 1893, *The Final Problem*

When Pure Strategies Won't Work

When Pure Strategies Won't Work

		Moriarty: Dover	Moriarty: Canterbury
Holmes	Dover	-1, 1	1, -1
Holmes	Canterbury	1, -1	-1, 1

Expected Value

- **Expected value** of a random variable X, written $E(X)$ (and sometimes μ), is the long-run average value of X "expected" after many repetitions

$$E(X) = \sum_{i=1}^{k} p_i x_i$$

- $E(X) = p_1 x_1 + p_2 x_2 + \cdots + p_k x_k$

- A **probability-weighted average** of X, with each possible X value, x_i, weighted by its associated probability p_i

- Also called the **"mean"** or **"expectation"** of X, always denoted either $E(X)$ or μ_X

Expected Value: Example I

Example: Suppose you lend your friend \$100 at 10% interest. If the loan is repaid, you receive \$110. You estimate that your friend is 99% likely to repay, but there is a default risk of 1% where you get nothing. What is the expected value of repayment?

Mixed Strategies

- **Pure strategy**: is a complete strategy profile that a player will play
    - Recall, **strategy** is a list of choices player will take at every possible decision node
- **Mixed strategy** is a **probability distribution** over a strategy profile
    - Plays a variety of pure strategies according to probabilities

Mixed Strategies

- The logic of mixed strategies is best understood in the context of repeated constant-sum games
- If you play one strategy repeatedly (i.e. a **pure strategy**), your opponent can exploit your (predictable) strategy with their best response
- You want to “keep your opponent guessing”

Mixed Strategy Nash Equilibrium

- We have already seen Nash equilibrium in pure strategies (PSNE)

- Nash (1950) proved that any n-player game with a finite number of pure strategies has at least one equilibrium
  - A game may have no PSNE, but there will always be a unique **mixed strategy Nash equilibrium (MSNE)**
  - Games may have *both* pure and a mixed NE

Mixed Strategy Nash Equilibrium

- Finding this is relatively straightforward with two players and two strategies

1. Let *p* be the probability of one player playing one of their available strategies
    - Let *(1-p)* be the probability of that player playing their other available strategy
2. Let *q* be the probability of the other player playing one of their available strategies
    - Let *(1-q)* be the probability of that player playing their other available strategy

- **There exists some *(p,q)* mix that is a Nash equilibrium in mixed strategies**

MSNE in Constant Sum Games

MSNE in Constant Sum Games

- Consider the following game between a **Kicker** and a **Goalie** during a penalty kick

MSNE in Constant Sum Games

- Consider the following game between a **Kicker** and a **Goalie** during a penalty kick

- A constant sum game (in this case, zero-sum)
    - If both choose same direction, **Goalie** blocks goal
    - If both choose different directions, **Kicker** gets goal

		Goalie	
		Dive Left	**Dive Right**
Kicker	**Kick Left**	-1 1	1 -1
	Kick Right	1 -1	-1 1

MSNE in Constant Sum Games

- Palacios-Huerta (2003) calculated average success rates in English, Spanish, & Italian leagues (1995-2000)
- If both **Kicker** and **Goalie** choose same direction, **Kicker**'s payoff is higher if he chooses his natural side (often Right)

		Goalie	
		Dive Left	Dive Right
Kicker	Kick Left	58, 42	95, 5
	Kick Right	93, 7	70, 30

Palacios-Huerta, Ignacio, 2003, "Professionals Play Minimax," *Review of Economic Studies* 70(2): 395–415

MSNE in Constant Sum Games

- This game has no Nash equilibrium in pure strategies (PSNE)
  - From any outcome, at least one player would prefer to switch strategies
  - No outcome has *all* players playing a best response

Kicker \ Goalie	Dive Left	Dive Right
Kick Left	58, <u>42</u>	<u>95</u>, 5
Kick Right	<u>93</u>, 7	70, <u>30</u>

MSNE in Constant Sum Games

- What if **Kicker** were to **randomize** strategies
  - Say 50% of the time, Kick Left, 50% of the time, Kick Right
- Let p be probability that **Kicker** plays Kick Left
  - $p = 0.50$

		Goalie: Dive Left	Goalie: Dive Right
Kicker	**Kick Left**	58, 42	95, 5
	Kick Right	93, 7	70, 30

MSNE in Constant Sum Games

- Then **Goalie** wants to maximize his **expected** payoff, given **Kicker** plays Kick Left with $p = 0.50$

		Goalie	
		Dive Left	**Dive Right**
Kicker	**Kick Left**	58, 42	95, 5
	Kick Right	93, 7	70, 30

MSNE in Constant Sum Games

- Then **Goalie** wants to maximize his **expected** payoff, given **Kicker** plays Kick Left with $p = 0.50$

- If **Goalie** plays Dive Left:

$$\mathbb{E}[\text{Dive Left}] = 42(p) + 7(1 - p)$$
$$= 42(0.50) + 7(1 - 0.50)$$

  - He can **expect** to earn 24.5

		Goalie	
		Dive Left	Dive Right
Kicker	Kick Left	58, 42	95, 5
	Kick Right	93, 7	70, 30

MSNE in Constant Sum Games

- Then **Goalie** wants to maximize his **expected** payoff, given **Kicker** plays Kick Left with $p = 0.50$

- If **Goalie** plays Dive Right:

$$\begin{aligned}\mathbb{E}[\text{Dive Right}] &= 5(p) + 30(1-p) \\ &= 5(0.50) + 30(1-0.50)\end{aligned}$$

  - He can **expect** to earn 17.5

		Goalie: Dive Left	Goalie: Dive Right
Kicker	Kick Left	58, 42	95, 5
	Kick Right	93, 7	70, 30

MSNE in Constant Sum Games

- Then **Goalie** wants to maximize his **expected** payoff, given **Kicker** plays Kick Left with $p = 0.50$

- **Goalie** will play Dive Left to maximize his expected payoff ($24.5 \succ 17.5$)

Goalie

Kicker		**Dive Left**	**Dive Right**
	Kick Left	58, 42	95, 5
	Kick Right	93, 7	70, 30

MSNE in Constant Sum Games

- Now consider **Kicker**'s **expected** payoff under this mixed strategy

- Since **Goalie** will Dive Left to maximize his expected payoff, **Kicker** can expect to earn:

$$58(p) + 93(1 - p)$$
$$58(0.50) + 93(1 - 0.50)$$
$$75.5$$

- **Goalie** playing Dive Left holds **Kicker**'s expected payoff down to 75.5

		Goalie	
		Dive Left	Dive Right
Kicker	Kick Left	58, 42	95, 5
	Kick Right	93, 7	70, 30

The Minimax Theorem

- In constant sum games, note that even in *mixed* strategies, one player increases their own (expected) payoff by pulling down the other player's (expected) payoff!

- In this game, even expected payoffs always sum to 100
    - **Kicker**'s $\mathbb{E}[\pi] = 75.5$
    - **Goalie**'s $\mathbb{E}[\pi] = 24.5$

The Minimax Theorem

- von Neumann & Morgenstern's **minimax theorem** (simplified): in a 2-person, constant sum game, each player maximizes their own expected payoff by minimizing their opponent's expected payoff
- The name **"minimax"** is a popular strategy in games, trying to minimize the risk of your maximum possible loss

Penalty Kicks: 50:50?

- **Kicker**'s “randomizing” 50:50 (Kick Left, Kick Right) was not random enough!
- **Goalie** recognizing this pattern can exploit it and hold down **Kicker**'s expected payoff
- **Kicker** can do better by picking a better p (and similarly, so can **Goalie**)
  - Hint: if **Goalie** knew **Kicker**'s p before **Goalie** chose, would he have a clearly better choice of Dive Left vs. Dive Right?

The Opponent Indifference Principle

- Want to find the optimal probability mix that **leaves your opponent(s) *indifferent* between their strategies to respond**

- In constant sum games (i.e. sports, board games, etc)

  - Making your opponent indifferent $\Longrightarrow$ minimizing your opponent's ability to recognize & exploit patterns in your actions

- This principle is the same in non-constant sum games too!

- Implies game is played repeatedly

- Not always intuitive, but a simple principle

Kicker's Optimal Choice of p

- We want to find **Kicker**'s optimal mixed strategy that leaves **Goalie** indifferent between his (pure) strategies
- Suppose **Kicker** plays Kick Left with probability p

		Goalie	
		Dive Left	**Dive Right**
Kicker	**Kick Left**	58, 42	95, 5
	Kick Right	93, 7	70, 30

Kicker's Optimal Choice of p

- We want to find **Kicker**'s optimal mixed strategy that leaves **Goalie** indifferent between his (pure) strategies
- Suppose **Kicker** plays Kick Left with probability p
- **Goalie**'s expected payoff of playing Dive Left: 42p+7(1-p)

		Goalie	
		Dive Left	Dive Right
Kicker	Kick Left	58, 42	95, 5
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Kicker's Optimal Choice of p

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- Suppose **Kicker** plays Kick Left with probability p
- **Goalie**'s expected payoff of playing Dive Left: 42p+7(1-p)
- **Goalie**'s expected payoff of playing Dive Right: 5p+30(1-p)

		Goalie	
		Dive Left	**Dive Right**
Kicker	**Kick Left**	58, 42	95, 5
	Kick Right	93, 7	70, 30
	***p*-mix**	42p-7(1-p)	5p+30(1-p)

Kicker's Optimal Choice of p

- We want to find **Kicker**'s optimal mixed strategy that leaves **Goalie** indifferent between his (pure) strategies

- Suppose **Kicker** plays Kick Left with probability p

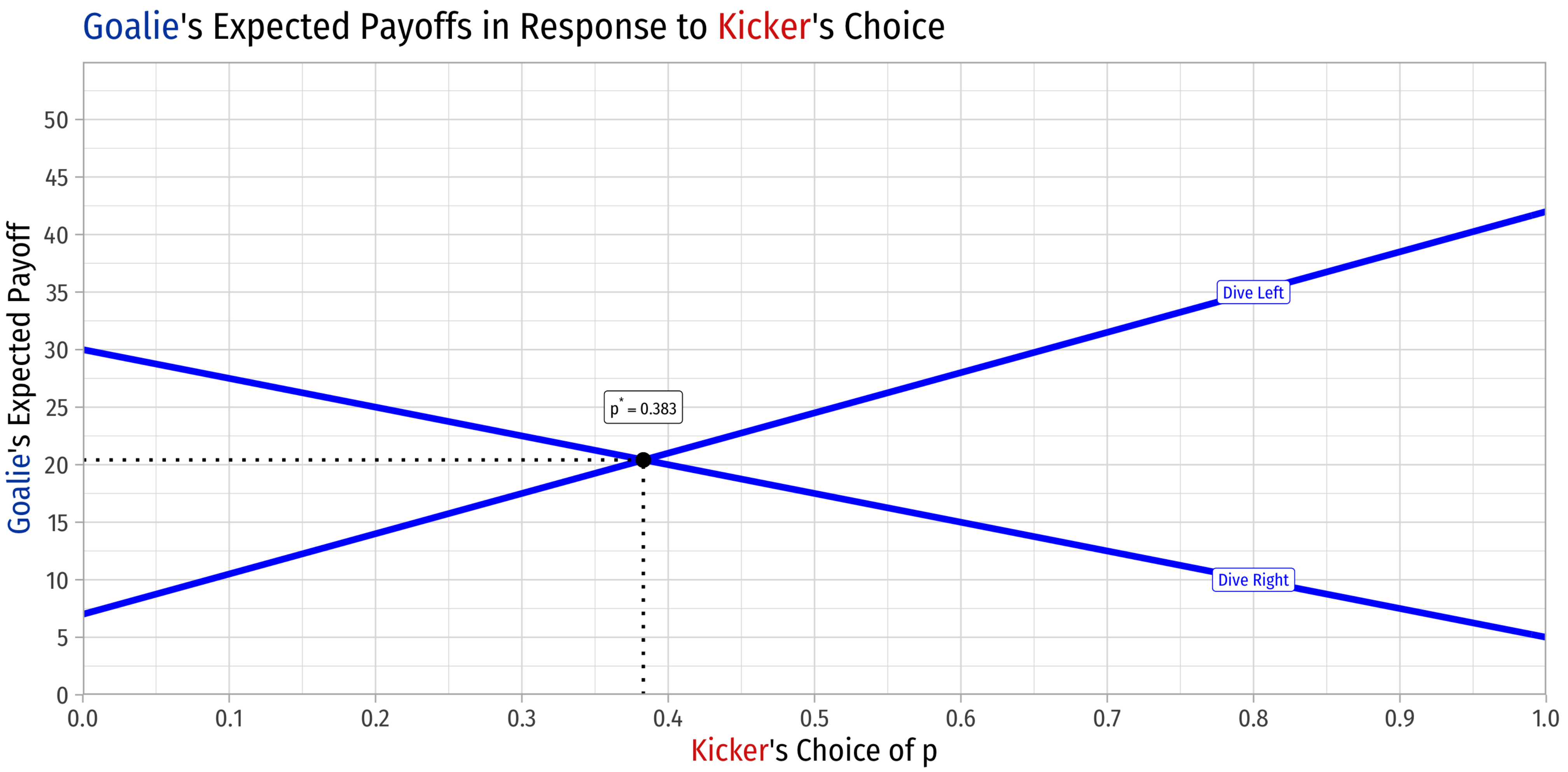
- **Goalie**'s expected payoff of playing Dive Left: 42p+7(1-p)

- **Goalie**'s expected payoff of playing Dive Right: 5p+30(1-p)

- What value of p would make **Goalie** indifferent between Dive Left and Dive Right?
    - i.e. $\mathbb{E}[Left] = \mathbb{E}[Right]$

		Goalie	
		Dive Left	**Dive Right**
Kicker	**Kick Left**	58, 42	95, 5
	Kick Right	93, 7	70, 30
	***p*-mix**	42p-7(1-p)	5p+30(1-p)

Kicker's Optimal Choice of p, Graphically



Kicker's Optimal Choice of p: Algebraically

- Find value of p that equates **Goalie**'s expected payoff of Dive Left and Dive Right:

$$\mathbb{E}[Left] = \mathbb{E}[Right]$$
$$\mathbb{E}[42p + 7(1-p)] = \mathbb{E}[5p + 30(1-p)]$$

- $p^\star = 0.383$

- **Kicker** plays Kick Left with $p = 0.383$ and Kick Right with $1 - p = 0.617$
  - **Goalie**'s expected payoff of Dive Left: $42(0.383) + 7(0.617) \approx 20.41$
  - **Goalie**'s expected payoff of Dive Right: $5(0.383) + 30(0.617) \approx 20.41$

Goalie's Optimal Choice of q

- We want to find **Goalie**'s optimal mixed strategy that leaves **Kicker** indifferent between his (pure) strategies

- Suppose **Goalie** plays Dive Left with probability q

		Goalie	
		Dive Left	**Dive Right**
Kicker	**Kick Left**	58, 42	95, 5
	Kick Right	93, 7	70, 30
	***p*-mix**	42p-7(1-p)	5p+30(1-p)

Goalie's Optimal Choice of q

- We want to find **Goalie**'s optimal mixed strategy that leaves **Kicker** indifferent between his (pure) strategies
- Suppose **Goalie** plays Dive Left with probability q
- **Kicker**'s expected payoff of playing Dive Left: 58q+95(1-q)

		Goalie	
		Dive Left	**Dive Right**
Kicker	**Kick Left**	58, 42	95, 5
	Kick Right	93, 7	70, 30
	***p*-mix**	42p-7(1-p)	5p+30(1-p)

Goalie's Optimal Choice of q

- We want to find **Goalie**'s optimal mixed strategy that leaves **Kicker** indifferent between his (pure) strategies
- Suppose **Goalie** plays Dive Left with probability q
- **Kicker**'s expected payoff of playing Dive Left: 58q+95(1-q)
- **Kicker**'s expected payoff of playing Dive Right: 93q+70(1-q)

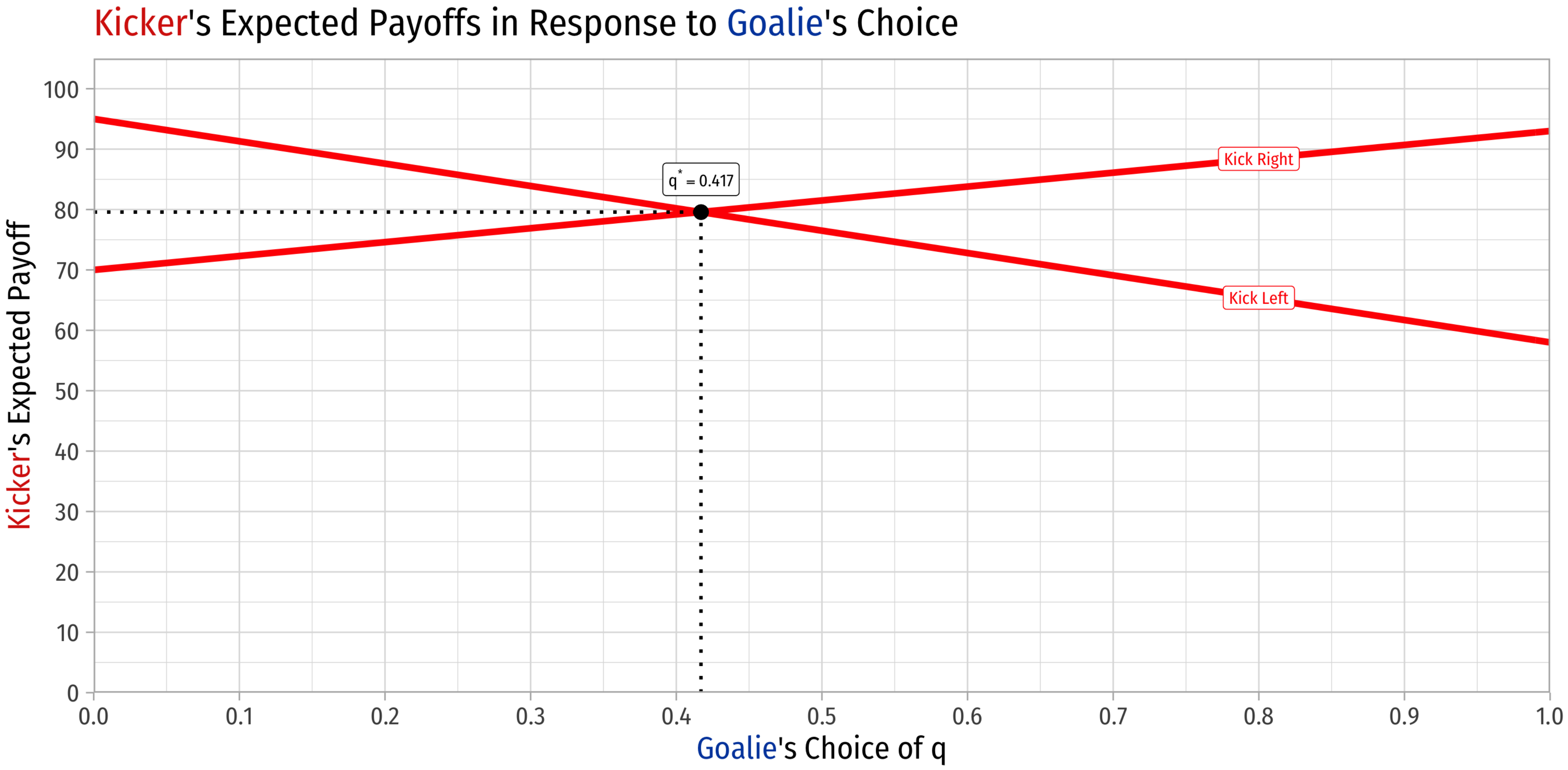
		Goalie		
		Dive Left	Dive Right	*q*-mix
Kicker	Kick Left	58, 42	95, 5	58q+95(1-q)
	Kick Right	93, 7	70, 30	93q+70(1-q)
	p-mix	42p-7(1-p)	5p+30(1-p)	

Goalie's Optimal Choice of q

- We want to find **Goalie**'s optimal mixed strategy that leaves **Kicker** indifferent between his (pure) strategies
- Suppose **Goalie** plays Dive Left with probability q
- **Kicker**'s expected payoff of playing Dive Left: 58q+95(1-q)
- **Kicker**'s expected payoff of playing Dive Right: 93q+70(1-q)
- What value of p would make **Kicker** indifferent between Kick Left and Kick Right?
  - i.e. $\mathbb{E}[Left] = \mathbb{E}[Right]$

		Goalie		
		Dive Left	**Dive Right**	***q*-mix**
Kicker	**Kick Left**	58, 42	95, 5	58q+95(1-q)
	Kick Right	93, 7	70, 30	93q+70(1-q)
	***p*-mix**	42p-7(1-p)	5p+30(1-p)	

Goalies's Optimal Choice of q, Graphically



Goalie's Optimal Choice of q: Algebraically

- Find value of q that equates **Kicker**'s expected payoff of Kick Left and Kick Right:

$$\mathbb{E}[Left] = \mathbb{E}[Right]$$
$$\mathbb{E}[58q + 95(1-q)] = \mathbb{E}[93q + 70(1-q)]$$

- $q^\star = 0.417$

- **Goalie** plays Dive Left with $q = 0.417$ and Dive Right with $1 - q = 0.583$
  - **Kicker**'s expected payoff of Kick Left: $58(0.417) + 95(0.583) \approx 79.57$
  - **Kicker**'s expected payoff of Kick Right: $93(0.417) + 70(0.583) \approx 79.57$

Mixed Strategy Nash Equilibrium

- **Goalie** is indifferent between Dive Left and Dive Right when **Kicker** plays Kick Left with $p=0.383$

- **Kicker** is indifferent between Kick Left and Kick Right when **Goalie** plays Dive Left with $q=0.417$

		Goalie		
		Dive Left	**Dive Right**	***q*-mix**
Kicker	**Kick Left**	58, 42	95, 5	$58q+95(1-q)$
	Kick Right	93, 7	70, 30	$93q+70(1-q)$
	***p*-mix**	$42p-7(1-p)$	$5p+30(1-p)$	(p^*,q^*)

Mixed Strategy Nash Equilibrium

- **Goalie** is indifferent between Dive Left and Dive Right when **Kicker** plays Kick Left with p=0.383

- **Kicker** is indifferent between Kick Left and Kick Right when **Goalie** plays Dive Left with q=0.417

- **Mixed Strategy Nash Equilibrium (MSNE)**: (p, q) = (0.383, 0.417)

		Goalie		
		Dive Left	**Dive Right**	***q*-mix**
Kicker	**Kick Left**	58, 42	95, 5	58q+95(1-q)
	Kick Right	93, 7	70, 30	93q+70(1-q)
	***p*-mix**	42p-7(1-p)	5p+30(1-p)	(p*,q*)

Mixed Strategy Nash Equilibrium

- **Goalie** is indifferent between Dive Left and Dive Right when **Kicker** plays Kick Left with p=0.383
- **Kicker** is indifferent between Kick Left and Kick Right when **Goalie** plays Dive Left with q=0.417
- **Mixed Strategy Nash Equilibrium (MSNE)**: $(p, q) = (0.383, 0.417)$
  - **Kicker**'s expected payoff: 79.57
  - **Goalie**'s expected payoff: 20.41
  - Note they sum to 1!

		Goalie		
		Dive Left	Dive Right	q-mix
Kicker	Kick Left	58, 42	95, 5	$58q+95(1-q)$
	Kick Right	93, 7	70, 30	$93q+70(1-q)$
	p-mix	$42p-7(1-p)$	$5p+30(1-p)$	(p^*,q^*)

p and q as Best Responses

- p = pr(Kicker kicks Left)
- q = pr(Goalie dives Left)

		Goalie		
		Dive Left	Dive Right	*q*-mix
Kicker	Kick Left	58, 42	95, 5	58q+95(1-q)
	Kick Right	93, 7	70, 30	93q+70(1-q)
	p-mix	42p-7(1-p)	5p+30(1-p)	(p*,q*)

p and q as Best Responses

- p = pr(Kicker kicks Left)
- q = pr(Goalie dives Left)
- **Goalie**'s Best Response

$$= \begin{cases} Right & \text{if } p < 0.383 \\ Indifferent & \text{if } p = 0.383 \\ Left & \text{if } p > 0.383 \end{cases}$$

		Goalie		
		Dive Left	**Dive Right**	***q*-mix**
Kicker	**Kick Left**	58, 42	95, 5	58q+95(1-q)
	Kick Right	93, 7	70, 30	93q+70(1-q)
	***p*-mix**	42p-7(1-p)	5p+30(1-p)	(p*,q*)

p and q as Best Responses

- p = pr(Kicker kicks Left)
- q = pr(Goalie dives Left)
- **Goalie**'s Best Response

$$= \begin{cases} Right & \text{if } p < 0.383 \\ Indifferent & \text{if } p = 0.383 \\ Left & \text{if } p > 0.383 \end{cases}$$

- **Kicker**'s Best Response

$$= \begin{cases} Left & \text{if } q < 0.417 \\ Indifferent & \text{if } q = 0.417 \\ Right & \text{if } q > 0.417 \end{cases}$$

		Goalie		
		Dive Left	**Dive Right**	***q*-mix**
Kicker	**Kick Left**	58, 42	95, 5	58q+95(1-q)
	Kick Right	93, 7	70, 30	93q+70(1-q)
	***p*-mix**	42p-7(1-p)	5p+30(1-p)	(p*,q*)

p and q as Best Responses

- p = pr(Kicker kicks Left)
- q = pr(Goalie dives Left)
- **Goalie**'s Best Response

$$= \begin{cases} Right & \text{if } p < 0.383 \\ Indifferent & \text{if } p = 0.383 \\ Left & \text{if } p > 0.383 \end{cases}$$

- **Kicker**'s Best Response

$$= \begin{cases} Left & \text{if } q < 0.417 \\ Indifferent & \text{if } q = 0.4173 \\ Right & \text{if } q > 0.417 \end{cases}$$

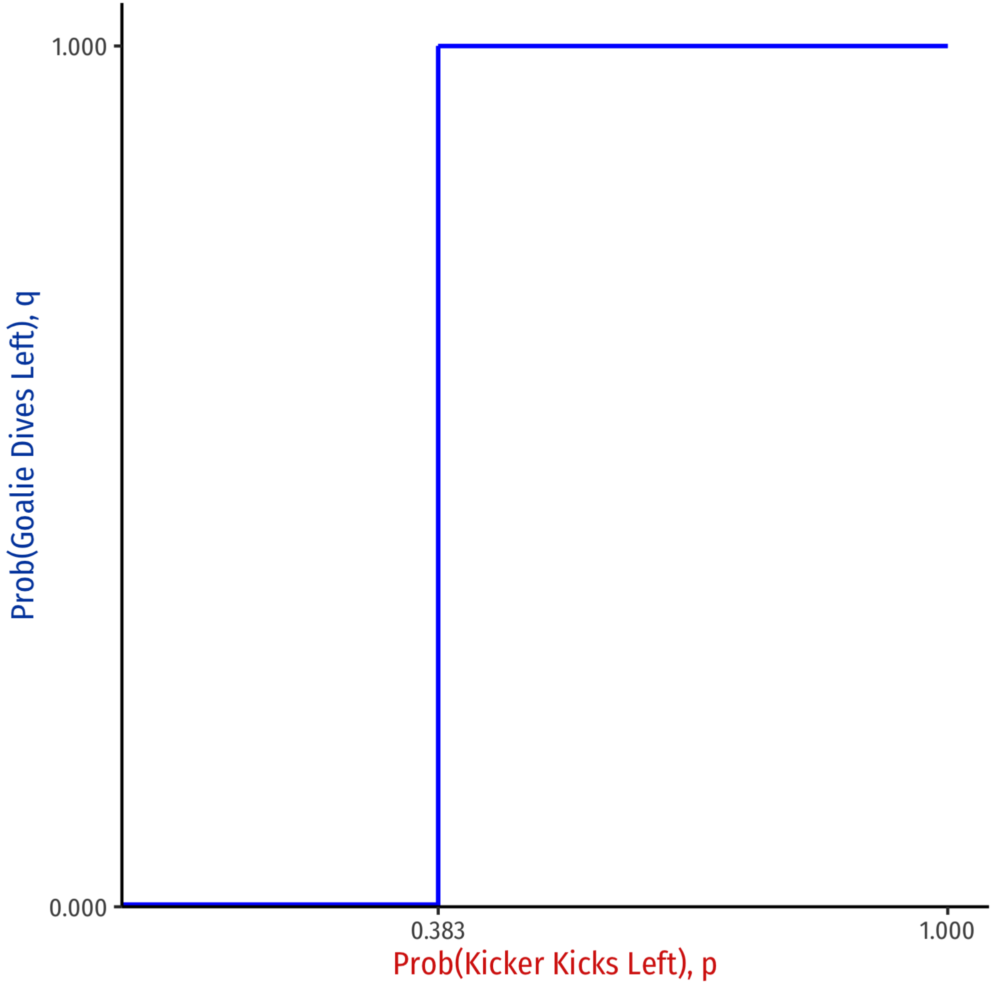
- Like any Nash equilibrium, players are playing mutual best responses to each other (probabilistically)

		Goalie		
		Dive Left	**Dive Right**	***q*-mix**
Kicker	**Kick Left**	58, 42	95, 5	58q+95(1-q)
	Kick Right	93, 7	70, 30	93q+70(1-q)
	***p*-mix**	42p-7(1-p)	5p+30(1-p)	(p*,q*)

Goalie's Best Reponse (q) to p

Goalie's Best Response

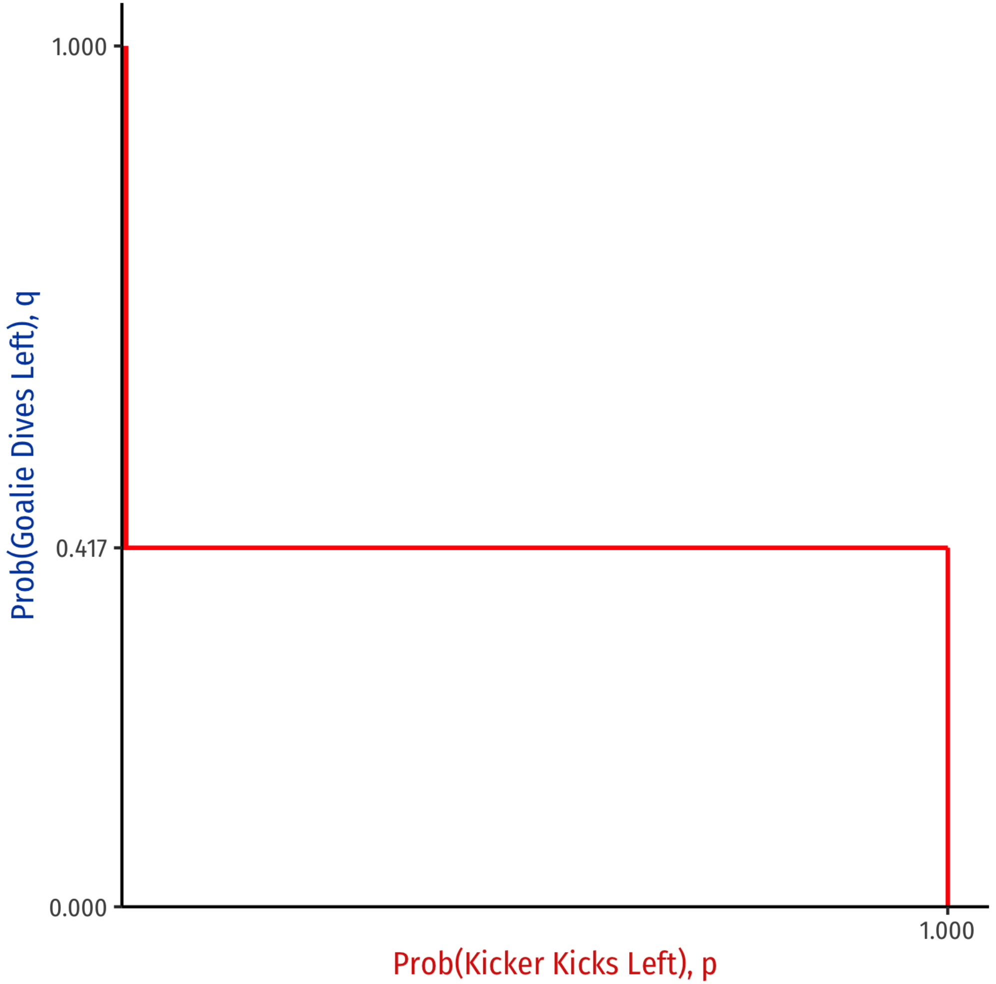
$$= \begin{cases} Right & \text{if } p < 0.383 \\ Indifferent & \text{if } p = 0.383 \\ Left & \text{if } p > 0.383 \end{cases}$$



Kicker's Best Reponse (p) to q

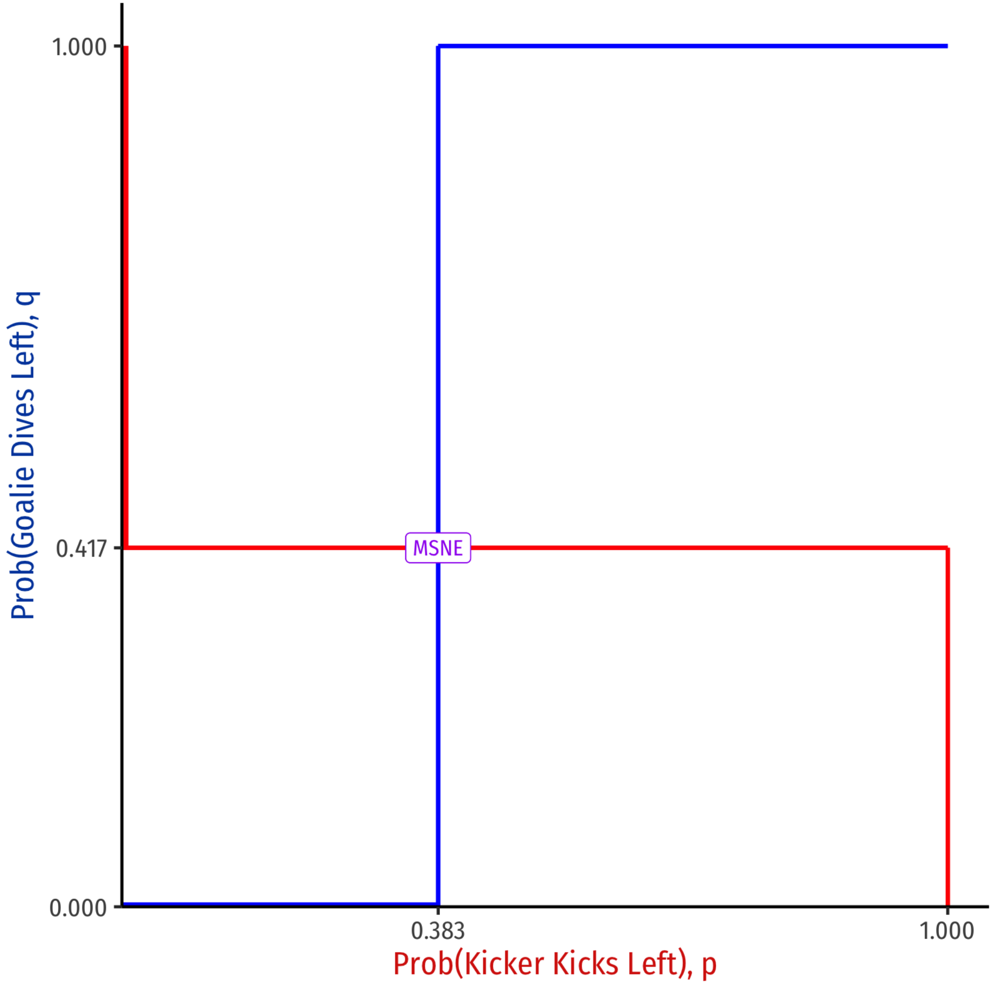
- **Kicker**'s Best Response

$$= \begin{cases} Left & \text{if } q < 0.417 \\ Indifferent & \text{if } q = 0.4173 \\ Right & \text{if } q > 0.417 \end{cases}$$



Mixed Strategy Nash Equilibrium

- Like any Nash equilibrium, where best response functions intersect



Rock-Paper-Scissors I

- A two player game with *three* strategies available to each
- Graphically more difficult, but same principle to find **MSNE**
    - find probabilities that make opponent indifferent between their responses
- Game is symmetric, so only need to find one player's optimal mixed strategy

		Column		
		Rock	**Paper**	**Scissors**
Row	**Rock**	0 0	-1 1	1 -1
	Paper	1 -1	0 0	-1 1
	Scissors	-1 1	1 -1	0 0

Rock-Paper-Scissors II

- Define for **Column**:
  - $r = \mathrm{pr}(\text{Rock})$
  - $p = \mathrm{pr}(\text{Paper})$
  - $1 - r - p = \mathrm{pr}(\text{Scissors})$

		Column		
		Rock	**Paper**	**Scissors**
Row	**Rock**	0, 0	-1, 1	1, -1
	Paper	1, -1	0, 0	-1, 1
	Scissors	-1, 1	1, -1	0, 0

Rock-Paper-Scissors II

- Define for **Column**:
  - $r =$ pr(Rock)
  - $p =$ pr(Paper)
  - $1 - r - p$ = pr(Scissors)
- **Column** must choose r, p that make **Row** indifferent between their strategies

		Column		
		Rock	Paper	Scissors
Row	Rock	0, 0	-1, 1	1, -1
	Paper	1, -1	0, 0	-1, 1
	Scissors	-1, 1	1, -1	0, 0

Rock-Paper-Scissors II

- List the expected payoffs to **Row** from **Column**'s mix of r, p
- **Row**'s expected payoff must equal for all three strategies
  - So let's take any two and set them equal:

$$2r + p - 1 = p - r$$

		Column			
		Rock	Paper	Scissors	Mix
	Rock	0, 0	-1, 1	1, -1	1-r-2p
Row	Paper	1, -1	0, 0	-1, 1	2r+p-1
	Scissors	-1, 1	1, -1	0, 0	p-r

Rock-Paper-Scissors II

- List the expected payoffs to **Row** from **Column**'s mix of r, p
- **Row**'s expected payoff must equal for all three strategies
  - So let's take any two and set them equal:

$$2r + p - 1 = p - r$$

- $r = \frac{1}{3}$
- $p = \frac{1}{3}$
- $(1 - r - p) = \frac{1}{3}$

		Column: Rock	Column: Paper	Column: Scissors	Column: Mix
Row	**Rock**	0, 0	-1, 1	1, -1	1-r-2p
	Paper	1, -1	0, 0	-1, 1	2r+p-1
	Scissors	-1, 1	1, -1	0, 0	p-r

Rock-Paper-Scissors II

- **MSNE**: each player plays all three strategies with equal probability $\left(\frac{1}{3}\right)$

		Column			
		Rock	Paper	Scissors	Mix
Row	Rock	0, 0	-1, 1	1, -1	1-r-2p
	Paper	1, -1	0, 0	-1, 1	2r+p-1
	Scissors	-1, 1	1, -1	0, 0	p-r

Coordination Games: PSNE and MSNE

MSNE in Coordination Games

- The necessity of MSNE is easy to see for constant-sum games with no PSNE
- But MSNE also exist for non-constant sum games, and for games with one or more PSNE

Assurance Game: MSNE

- We know an **assurance game** has two PSNE
- Let's solve for MSNE

		Sally: Whitaker	Sally: Starbucks
Harry	Whitaker	2, 2	0, 0
Harry	Starbucks	0, 0	1, 1

Assurance Game: MSNE

- Let p =pr(**Harry** goes to Whitaker)
- Let q =pr(**Sally** goes to Whitaker)

		Sally: Whitaker	Sally: Starbucks
Harry	Whitaker	2, 2	0, 0
	Starbucks	0, 0	1, 1

Assurance Game: MSNE

- Let p =pr(**Harry** goes to Whitaker)
- Let q =pr(**Sally** goes to Whitaker)

		Sally		
		Whitaker	**Starbucks**	***q*-mix**
Harry	**Whitaker**	2, 2	0, 0	2q
	Starbucks	0, 0	1, 1	1-q
	***p*-mix**	2p	1-p	(p*,q*)

Assurance Game: MSNE

- Let p =pr(**Harry** goes to Whitaker)
- Let q =pr(**Sally** goes to Whitaker)
- $p^\star = \frac{1}{3}$
- $q^\star = \frac{1}{3}$
- **MSNE**: (p, q) = $(\frac{1}{3}, \frac{1}{3})$

		Sally		
		Whitaker	**Starbucks**	***q*-mix**
Harry	**Whitaker**	2, 2	0, 0	2q
	Starbucks	0, 0	1, 1	1-q
	***p*-mix**	2p	1-p	(p*,q*)

Assurance Game: MSNE

- Calculate expected payoffs to **Harry** and **Sally** with (p, q) MSNE

		Sally		
		Whitaker	Starbucks	*q*-mix
Harry	Whitaker	2, 2	0, 0	2q
	Starbucks	0, 0	1, 1	1-q
	p-mix	2p	1-p	(p*,q*)

Assurance Game: MSNE

- Calculate expected payoffs to **Harry** and **Sally** with (p, q) MSNE
  - **Harry**: $\frac{2}{3}$
  - **Sally**: $\frac{2}{3}$

		Sally: Whitaker	Sally: Starbucks	*q*-mix
Harry	**Whitaker**	2, 2	0, 0	2q
Harry	**Starbucks**	0, 0	1, 1	1-q
	***p*-mix**	2p	1-p	(p*,q*)

Assurance Game: MSNE

- Calculate expected payoffs to **Harry** and **Sally** with (p, q) MSNE
  - **Harry**: $\frac{2}{3}$
  - **Sally**: $\frac{2}{3}$
- Problem: MSNE is even worse than either PSNE in this game!
  - Significant probability of going to different places
  - Also very fragile, anything $>, <\ \frac{2}{3}$ reverts to PSNE

		Sally		
		Whitaker	**Starbucks**	***q*-mix**
Harry	**Whitaker**	2, 2	0, 0	2q
	Starbucks	0, 0	1, 1	1-q
	***p*-mix**	2p	1-p	(p*,q*)

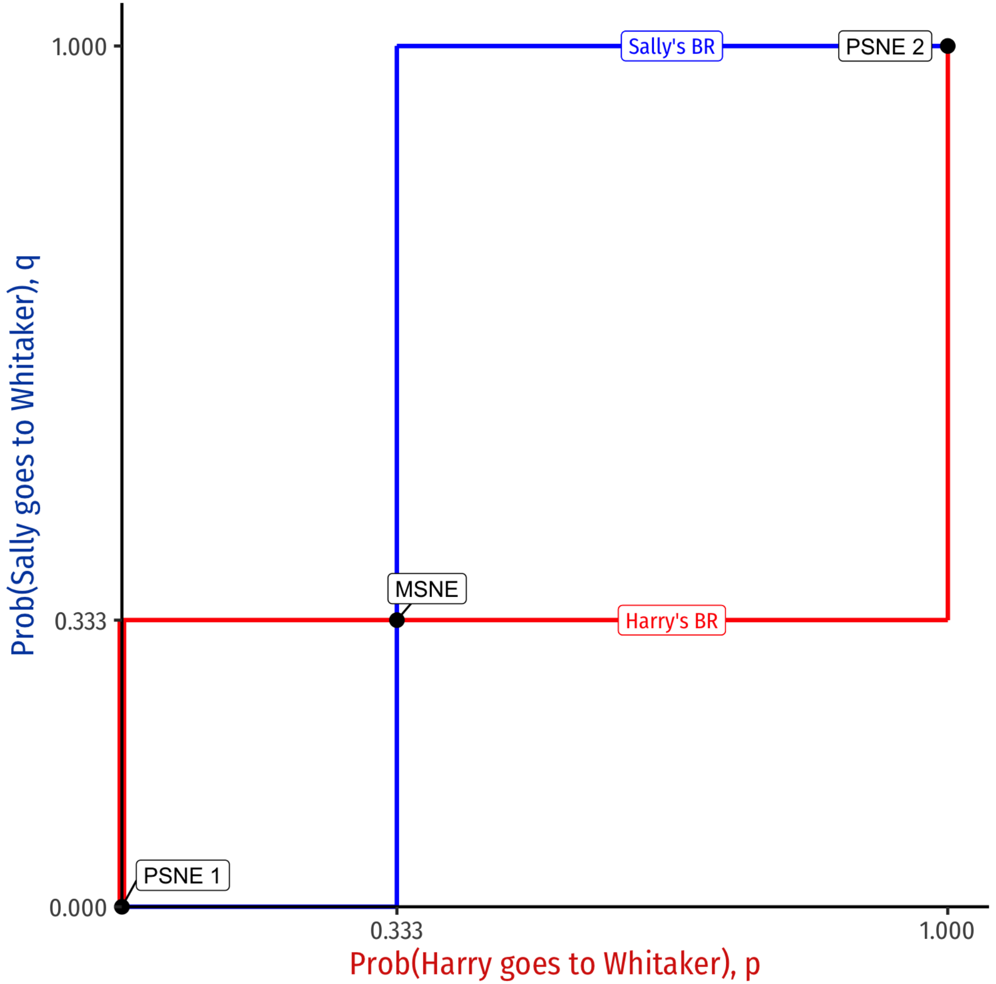
Assurance Game: MSNE

- **Sally's BR**

$$= \begin{cases} Starbucks & \text{if } p < \frac{1}{3} \\ Indifferent & \text{if } p = \frac{1}{3} \\ Whitaker & \text{if } p > \frac{1}{3} \end{cases}$$

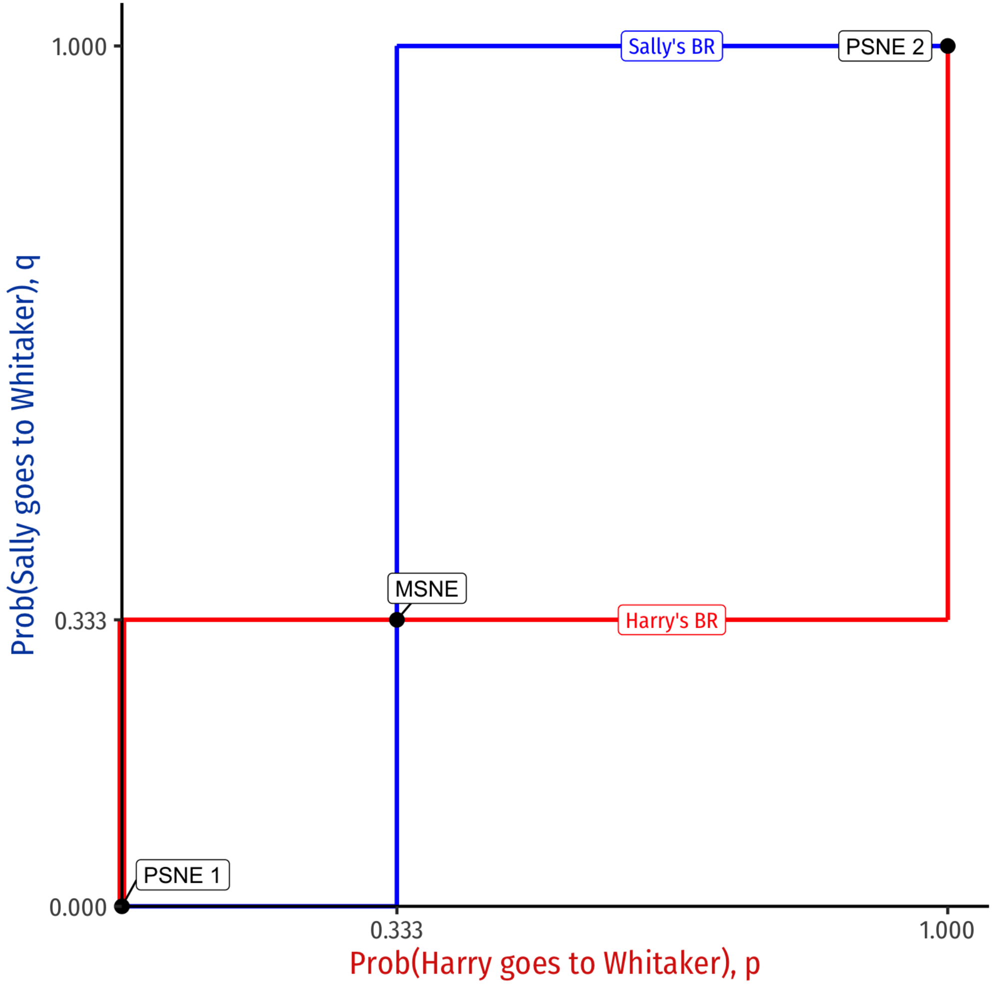
- **Harry's BR**

$$= \begin{cases} Starbucks & \text{if } q < \frac{1}{3} \\ Indifferent & \text{if } q = \frac{1}{3} \\ Whitaker & \text{if } q > \frac{1}{3} \end{cases}$$



Assurance Game: MSNE

- All intersections of best response functions are Nash equilibria
- Interior solution: MSNE
- Corner solutions: PSNE
    - PSNE are special cases of MSNE where $p \in \{0, 1\}$ and $q \in \{0, 1\}$



Chicken/Hawk-Dove Game: MSNE

- Hawk-Dove/Chicken game: 2 PSNE
- Let's solve for MSNE

		Column	
		Hawk	Dove
Row	Hawk	-1, -1	2, 0
	Dove	0, 2	1, 1

Chicken/Hawk-Dove Game: MSNE

- Let p =pr(**Row** plays Hawk)
- Let q =pr(**Column** plays Hawk)

		Column Hawk	**Column** Dove
Row	**Hawk**	-1, -1	2, 0
Row	**Dove**	0, 2	1, 1

Chicken/Hawk-Dove Game: MSNE

- Let p =pr(**Row** plays Hawk)
- Let q =pr(**Column** plays Hawk)

		Column		
		Hawk	**Dove**	***q*-mix**
Row	**Hawk**	-1, -1	2, 0	2-3q
	Dove	0, 2	1, 1	1-q
	***p*-mix**	2-3p	1-p	(p*,q*)

Chicken/Hawk-Dove Game: MSNE

- Let p =pr(**Row** plays Hawk)
- Let q =pr(**Column** plays Hawk)
- $p^\star = 0.5$
- $q^\star = 0.5$
- **MSNE**: (p, q) = $(0.5, 0.5)$

Column

Row

	Hawk	Dove	*q*-mix
Hawk	-1, -1	2, 0	2-3q
Dove	0, 2	1, 1	1-q
***p*-mix**	2-3p	1-p	(p*,q*)

Chicken/Hawk-Dove Game: MSNE

- Calculate expected payoffs to **Row** and **Column** with (p, q) MSNE

Column

Row		Hawk	Dove	*q*-mix
	Hawk	-1, -1	2, 0	2-3q
	Dove	0, 2	1, 1	1-q
	***p*-mix**	2-3p	1-p	(p*,q*)

Chicken/Hawk-Dove Game: MSNE

- Calculate expected payoffs to **Row** and **Column** with (p, q) MSNE
  - **Row**: 0.5
  - **Column**: 0.5

		Column		
		Hawk	**Dove**	***q*-mix**
Row	**Hawk**	-1, -1	2, 0	2-3q
	Dove	0, 2	1, 1	1-q
	***p*-mix**	2-3p	1-p	(p*,q*)

Chicken/Hawk-Dove Game: MSNE

- Calculate expected payoffs to **Row** and **Column** with (p, q) MSNE
  - **Row**: 0.5
  - **Column**: 0.5
- Expected payoff in MSNE is:
  - better than PSNE when you're a **dove** against a **hawk**
  - worse than PSNE when you're a **hawk** against a **dove**

		Column		
		Hawk	**Dove**	***q*-mix**
Row	**Hawk**	-1, -1	2, 0	2-3q
	Dove	0, 2	1, 1	1-q
	***p*-mix**	2-3p	1-p	(p*,q*)

Chicken/Hawk-Dove Game: MSNE

- **Column's BR**

$$= \begin{cases} Hawk & \text{if } p < 0.5 \\ Indifferent & \text{if } p = 0.5 \\ Dove & \text{if } p > 0.5 \end{cases}$$

- **Row's BR**

$$= \begin{cases} Hawk & \text{if } q < 0.5 \\ Indifferent & \text{if } q = 0.5 \\ Dove & \text{if } q > 0.5 \end{cases}$$